40.13 Room temperature atmospheric air flows at $100\frac{ft}{s}$ in a $400ft$ long nominal $3in$ schedule 40 threaded steel pipe containing (12) 90° long radius elbows and (2) gate valves. Air leaves the pipe at an elevation $200ft$ lower than the pipe inlet. What is the pressure difference between the two ends of the pipe?

A. $3psi$

B. $18psi$

C. $96psi$

D. $110psi$

Start by writing the Bernoulli Equation:

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f$$

Neglect velocity and solve for the pressure difference $\Delta P = P_1 - P_2$. Note $\Delta z = z_2 - z_1 = -200ft$ which is a negative value since the entrance is higher than the exit.

$$\frac{P_1 - P_2}{\gamma} = (z_2 - z_1) + h_f$$

The friction losses include both major and minor losses. This requires an additional term beyond the typical application of the Darcy Equation, as can be found by looking up Fittings Losses in the reference handbook. You may factor out the common terms:

$$h_f = h_{f,major} + h_{f,minor} = \frac{fLv^2}{2gD} + K\frac{v^2}{2g} = \left(\frac{fL}{D} + K\right)\left(\frac{v^2}{2g}\right)$$

For the minor losses, look up the K-factors for Threaded Pipe Fittings and take the sum accounting for the quantities:

Threaded Pipe Fittings	K-Factor	Total
(12) 90° long radius elbow	.31	3.72
(2) gate valve	.14	.28

Taking the overall sum, $K_{total} = 4$

For the major losses, using the Steel Pipe Friction Tables to find the actual diameter of a nominal $3inch$ pipe. Use the Properties of Air table to find the kinematic viscosity at room temperature. Note the velocity is given.

$$D = 3.068in\left(\frac{1ft}{12in}\right) = .264ft$$

$$Re = \frac{vD}{\nu} = \frac{\left(100\frac{ft}{s}\right)(.264ft)}{16.5 \times 10^{-5}\frac{ft^2}{s}} = 160,000 \approx 1.6 \times 10^5$$

Find the relative roughness:

$$\frac{\epsilon}{D} = \frac{.0002ft}{.264ft} \approx .0008$$

Find the friction factor using the Moody Diagram:

$$f = f\left(Re, \frac{\epsilon}{D}\right) \approx .021$$

Solve for the friction losses, including major and minor:

$$h_f = \left(\frac{fL}{D} + K\right)\left(\frac{v^2}{2g}\right) = \left(\frac{(.021)(400ft)}{(.264ft)} + 4\right)\left(\frac{\left(100\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)}\right) = 5559ft$$

This may seem high but recall the fluid is air. Calculate the pressure difference in feet of air and then convert to psi. Return to the Bernoulli Equation:

$$\frac{P_1 - P_2}{\gamma} = (z_2 - z_1) + h_f = (-200ft) + (5559ft) = 5359ft$$

Recall the specific weight has the same magnitude as density but has units of $\frac{lb_f}{ft^3}$ rather than $\frac{lb_m}{ft^3}$. Assume a typical density for room temperature air:

$$\gamma_{air} = \frac{\rho_{air} \cdot g}{g_c} = \frac{\left(.075\frac{lb_m}{ft^3}\right)\left(32.2\frac{ft}{s^2}\right)}{\left(32.2\frac{lb_m}{lb_f}\frac{ft}{s^2}\right)} = .075\frac{lb_f}{ft^3}$$

Solve for the pressure difference in psi:

$$\Delta P = (P_1 - P_2) = (5359ft)(\gamma_{air})$$

$$\Delta P = (5359ft)\left(.075\frac{lb_f}{ft^3}\right) = 417\frac{lb_f}{ft^2}\left(\frac{1ft^2}{144in^2}\right) = 2.9\frac{lb_f}{in^2}$$

Answer A