

40.23 An energy storage facility uses pumped hydro to capitalize on the low cost of off-peak electricity of \$0.02 per kWh. 400gpm of 60°F water is pumped from a low reservoir to a high reservoir at 1000ft higher elevation through an equivalent length of 3000ft of 5in schedule 40 steel pipe using a pump driven by an AC motor. The overall efficiency of the pump set is 75%. Assuming the off-peak operation lasts for 8hrs per night, what is the cost of electricity for one night?

- A. \$2
- B. \$13
- C. \$17
- D. \$52

The cost depends on the run time, the unit rate for electricity, and the electrical power demand, the latter which is unknown. The electrical power, P , depends on the hydraulic horsepower, whp , which can be determined using the volume flow rate, head added by the pump, and the efficiency. Note the efficiency given is the *total* efficiency including the pump losses and motor losses, yielding electrical power directly from hydraulic horsepower, skipping the intermediate step of specifying the brake horsepower, which is not a required step.

$$P = \frac{whp}{\eta} = \frac{Q_{[gpm]}\Delta h_{[ft]}}{3960\eta}$$

Write the form of the **Bernoulli Equation** suited for pumping applications:

$$\Delta h = h_A = \frac{P_2 - P_1}{\gamma} + \frac{v_2^2 - v_1^2}{2g} + z_2 - z_1 + h_f$$

Although the two reservoirs are at different elevations, presumably both are exposed to atmospheric pressure, and the column of air associated with the elevation difference does not have a meaningful impact on the static pressure. Therefore:

$$P_1 \approx P_2 \rightarrow \frac{P_2 - P_1}{\gamma} \approx 0$$

Since the volume of the reservoirs is large compared to the entering flow, the velocity term is also negligible:

$$v_1 \approx v_2 \approx 0$$

$$\frac{v_2^2 - v_1^2}{2g} \approx 0$$

The difference in height is given:

$$\Delta z = z_2 - z_1 = 1000ft$$

To find the losses, write the **Darcy Equation**:

$$h_f = \frac{fLv^2}{2gD}$$

Using the **Steel Pipe Friction Tables** for 400gpm flowing through a nominal 5inch standard weight steel pipe.

$$D = 5.047in \left(\frac{1ft}{12in} \right) = .4206ft$$

$$Q = 400gpm$$

$$v = 6.41 \frac{ft}{s}$$

Find the relative roughness:

$$\frac{\epsilon}{D} = \frac{.0002ft}{.4206ft} \approx .0005$$

Find the **Reynolds Number**. Use the **Properties of Water** table to look up the **kinematic viscosity**:

$$Re = \frac{vD}{\nu} = \frac{\left(6.41 \frac{ft}{s} \right) (.4206ft)}{1.217 \times 10^{-5} \frac{ft^2}{s}} = 220,000 \approx 2 \times 10^5$$

Use the **Moody Diagram** to find the friction factor:

$$f = f\left(Re, \frac{\epsilon}{D}\right) = .019$$

Substitute and solve the Darcy Equation:

$$h_f = \frac{fLv^2}{2gD} = \frac{(.019)(3000ft) \left(6.41 \frac{ft}{s} \right)^2}{2(.4206ft) \left(32.2 \frac{ft}{s^2} \right)} = 86.5ft$$

Determine the head added by the pump:

$$h_A = 1000ft + 86.5ft = 1086.5ft$$

Determine the electrical power demand and convert to KW:

$$P = \frac{whp}{\eta} = \frac{Q_{[gpm]}\Delta h_{[ft]}}{3960\eta} = \frac{(400)(1086.5)}{3960(.75)} = 146.3hp \left(\frac{.7457KW}{1hp} \right) = 109.1KW$$

Calculate the cost for one night of off-peak operation:

$$Cost = (109.1KW)(8hr) \left(\frac{\$.02}{KWH} \right) = \$17.46$$

Answer C