43.3 A $1000cfm$ fan coil unit installed for dehumidification maintains an apparatus dew point of $40°F$ and has a coil bypass factor of 10%. The entering air is $80°F$ dry bulb and 50% relative humidity. What quantity of total cooling is provided?

A. $3.2tons$

B. $3.6tons$

C. $5.4tons$

D. $6.0tons$

The bypass factor is the portion of the air flow that gets by the coil without being cooled. The bypass factor is the complement of the coil efficiency. When the bypass factor is 0, the coil is 100% efficient. In this case,

$$BF = 1 - \eta_{coil} = .1$$

$$\eta_{coil} = .9$$

The coil efficiency can be expressed as a ratio of the reduction in dry bulb temperature actually achieved as compared with what could possibly be achieved by the supply air temperature converging with the apparatus dew point i.e. the physical temperature of the coil. Let T_1 represent the entering air condition, T_2 represent the supply air condition, and ADP represent the coil condition. Write an expression for the coil efficiency:

$$\eta_{coil} = \frac{T_1 - T_2}{T_1 - T_{ADP}} = .9$$

Substituting known entering conditions and coil conditions, solve for the supply air temperature leaving the coil:

$$\frac{80 - T_2}{80 - 40} = .9 \rightarrow T_2 = 44°F$$

If the problem were asking for the quantity of sensible cooling provided, we would now have enough information since only the dry bulb temperatures would be needed. However, in order to find the total cooling provided, the supply air condition must be fully defined.

Use the psychrometric chart to look up the enthalpy and humidity ratio for the entering air:

$$T_1 = 80°F$$

$$\phi_1 = 50\%$$

$$h_1 = 31.2\frac{Btu}{lb}$$

$$\omega_1 = 76.4\frac{gr}{lb}$$

Look up the humidity ratio at the ADP:

$$T_{ADP} = 40^\circ F$$

$$\phi_{ADP} = 100\%$$

$$\omega_{ADP} = 36.3\frac{gr}{lb}$$

Since the humidity ratio changes linearly on the psychrometric chart for straight process line (as does dry bulb temperature), write an expression for the coil efficiency as a ratio of the *actual* change in humidity ratio achieved as compared to the maximum possible change if the process line terminated at the ADP (the final state would be on the saturation curve). Solve for the humidity ratio at the supply condition:

$$\eta_{coil} = \frac{\omega_1 - \omega_2}{\omega_1 - \omega_{ADP}} = .9$$

$$\frac{76.4\frac{gr}{lb} - \omega_2}{76.4\frac{gr}{lb} - 36.3\frac{gr}{lb}} = .9 \rightarrow \omega_2 = 40.3\frac{gr}{lb}$$

The supply air condition (state 2) is now fully defined. Use the psychrometric chart once more to look up the enthalpy at state 2:

$$T_2 = 44^\circ F$$

$$\omega_2 = 40.3\frac{gr}{lb}$$

$$h_2 = 16.8\frac{Btu}{lb}$$

Use the total cooling rule of thumb for air. Provided volume flow rate is in cfm and enthalpy is in $\frac{Btu}{lb}$, it is not necessary to write units. The result will have units of $\frac{Btu}{hr}$.

$$\dot{Q}_t = 4.5cfm\Delta h$$

$$\dot{Q}_t = 4.5\,(1000)\,(31.2 - 16.8) = 64,800\frac{Btu}{hr}$$

Convert to refrigeration tons:

$$\dot{Q}_t = 64,800\frac{Btu}{hr}\left(\frac{1\,ton}{12,000\frac{Btu}{hr}}\right) = 5.4\,tons$$

Answer C