43.10 The pressure loss in a duct is $1.2 in\,wg$ when the flow rate is $2500 cfm$. If the fan slows down to $2000 cfm$, what is the new pressure under reduced speed conditions?

A. $0.8 in\,wg$

B. $1.0 in\,wg$

C. $1.5 in\,wg$

D. $1.9 in\,wg$

Look up Fan Affinity Laws in the Reference Handbook and see the table called Fan Laws. Choose Law #3b:

$$P_1 = P_2 \times \left(\frac{D_2}{D_1}\right)^4 \left(\frac{Q_1}{Q_2}\right)^2 \left(\frac{\rho_1}{\rho_2}\right)$$

There is no indication that the fan diameter is being changed. Therefore assume $D_1 = D_2$. The density of the air under the new conditions may also be assumed to be unchanged. Therefore assume $\rho_1 = \rho_2$. Simplify:

$$P_1 = P_2 \times \left(\frac{Q_1}{Q_2}\right)^2$$

Note: It may be advisable to memorize some common uses of the fan laws where only one parameter is being adjusted. It is common to recall that pressure is a function of the volume flow rate squared.

For convenience, swap the subscripts and make a quick table to ensure proper handling of knowns and unknowns. Let condition 1 be the existing fan and condition 2 be for reduced speed.

$$P_2 = P_1 \times \left(\frac{Q_2}{Q_1}\right)^2$$

Parameter	Condition 1	Condition 2
$Q_{[cfm]}$	2500	2000
$P_{[in\,wg]}$	1.2	P_2

Solve for P_2 :

$$P_2 = 1.2 in\,wg \times \left(\frac{2000 cfm}{2500 cfm}\right)^2 = .77 in\,wg$$

Note there is a nonlinear reduction in pressure. This should be consistent with intuition as pressure changes with the square of the speed (and volume flow rate).

Answer A

43.11 An air handling fan delivers $9000 cfm$ against a total pressure of $3 in\, wg$ at standard conditions. A new unit is designed to provide the same volume flow using a fan with a 25% larger diameter which runs at half the speed. What is the air horsepower produced by the new air handler?

A. $1hp$

B. $2hp$

C. $4hp$

D. $5hp$

Make a table to organize the original and new conditions, and use subscripts 1 and 2, respectively. Take into account the given information regarding the new diameter increasing by 25% and the new speed reducing by half.

	Original (1)	New (2)
Q	$9000cfm$	$9000cfm$
P	$3in\, wg$	P_2
D	D_1	$D_2 = 1.25D_1$
N	N_1	$N_2 = \frac{N_1}{2}$
W	W_1	W_2

Look up Fan Power in the reference handbook and use the formula to determine the air horsepower for the original state, W_1. This book will typically use AHP; however, the handbook uses P_s or P_e, and the Fan Laws table uses W for power. In this problem we will freely interchange between all of these parameters with the understanding that all refer to *air horsepower*.

$$AHP_1 = W_1 = \frac{Q_1 \Delta P_1}{6356} = \frac{(9000)\,(3)}{6356} = 4.25hp$$

Note there is no reference to efficiency so assume $\eta = 1$ and may be ignored.

Select from the Fan Affinity Laws equation #1c:

$$W_1 = W_2 \times \left(\frac{D_1}{D_2}\right)^5 \left(\frac{N_1}{N_2}\right)^3 \left(\frac{\rho_1}{\rho_2}\right)$$

The density of the air under the new conditions may be assumed to be unchanged. Therefore assume $\rho_1 = \rho_2$. Swap the subscripts for convenience. Simplify and solve for the new power, W_2. Note the diameters and speeds need not be known. Instead the ratio of new to old may be used.

$$W_2 = W_1 \times \left(\frac{D_2}{D_1}\right)^5 \left(\frac{N_2}{N_1}\right)^3$$

$$W_2 = (4.25hp)\,(1.25)^5\,(.5)^3 = 1.62hp$$

There are alternative solutions to this problem using a different fan affinity law. The laws are internally consistent so if time permits, you may want to check your answer by taking a different avenue.

Answer B