43.18 Boston typically has 5,630 heating degree days in the winter based on a non-standard internal design temperature of $70°F$. A building in Boston designed for an inside temperature of $70°F$ and an outside temperature of $0°F$ loses $5\frac{MMBtu}{hr}$ on a design day. The building is heated with natural gas which has an average heating value of $1,000\frac{Btu}{ft^3}$ and an average cost of $0.90 per CCF. The boiler efficiency is 80%. What is the annual cost of heating?

A. $87,000

B. $94,000

C. $109,000

D. $117,000

Consider the units of degree days, $[°F \cdot days]$, i.e. $\Delta T \times time$. The rate of heat loss on a design day and the design ΔT are given. Write an expression for the quantity of heat lost during the heating season. Be sure the resulting units are in Btu.

$$\dot{Q} = \frac{Q}{t} \rightarrow Q = \dot{Q}t$$

There is no cookie-cutter formula for this problem. Multiplying the heating degree days with $\dot{Q}$ cancels time, leaving only energy and temperature. Dividing by the design ΔT eliminates temperature:

$$Q = \frac{HDD \times \dot{Q}}{\Delta T} = \frac{(5630 deg \cdot days)\left(\frac{24hr}{1day}\right)\left(5,000,000\frac{Btu}{hr}\right)}{(70°F - 0°F)} = 9.65 \times 10^9 Btu$$

Note that degree days account for the heat loss and ΔT being *less than* design for a great deal of the season. Heating systems are designed for the maximum (or near maximum) demand for the location, which may only be required on a few days per year.

Calculate the cost of natural gas. Note the heating value is given in $\frac{Btu}{ft^3}$ and the cost is in $\frac{\$}{CCF}$, where $CCF = 100ft^3$. Also account for the efficiency of the system, which will drive costs higher than if the system was 100% efficient.

$$Cost = \frac{\left(\frac{9.65\times10^9 Btu}{1000\frac{Btu}{ft^3}}\right)\left(\frac{\$0.90}{100ft^3}\right)}{(.8)} = \$108,579$$

Answer C

43.19 A house has a flat roof comprised of asphalt shingles on top of $2in$ of roof insulation on top of $3in$ of southern pine wood. Below the pinewood there is a $3^1/_2$ inch air gap followed by $^7/_8$ inch acoustical ceiling with density $21\frac{lb}{ft^3}$. The pine and the acoustical tiles both have an emissivity of 0.9. The outdoor design temperature is $50°F$ and the indoor design temperature is $70°F$. Assume $15mph$ wind. What is the rate of heat loss per unit area through the roof?

A. $1.5\frac{Btu}{hr \cdot ft^2}$

B. $1.7\frac{Btu}{hr \cdot ft^2}$

C. $1.8\frac{Btu}{hr \cdot ft^2}$

D. $2.0\frac{Btu}{hr \cdot ft^2}$

Calculate the resistance for each layer of the roof and combine to determine the total resistance.

For the outside surface, look up Surface Film Coefficients in the Reference Handbook:

$$R_o = \frac{1}{h_o} = \frac{1}{\left(6.00\frac{Btu}{hr \cdot ft^2 \cdot °F}\right)} = .17\frac{hr \cdot ft^2 \cdot °F}{Btu}$$

For the asphalt shingles, look up Building Materials or Roofing or Asphalt Shingles in the Reference Handbook and select the resistance for the thickness listed:

$$R_{shingles} = .44\frac{hr \cdot ft^2 \cdot °F}{Btu}$$

For the roof insulation, look up Building Materials or Roof Insulation in the Reference Handbook and select the resistance per inch of material thickness. Multiply by the thickness to obtain the resistance for the insulation:

$$R_{insulation} = \left(2.94\frac{hr \cdot ft^2 \cdot °F}{Btu \cdot in}\right)(2in) = 5.88\frac{hr \cdot ft^2 \cdot °F}{Btu}$$

For the southern pine, look up Building Materials or Softwoods or Southern Pine in the Reference Handbook and select the average resistance per inch of material thickness. Multiply by the thickness to obtain the resistance for the pine:

$$R_{pine} = \left(0.95\frac{hr \cdot ft^2 \cdot °F}{Btu \cdot in}\right)(3in) = 2.85\frac{hr \cdot ft^2 \cdot °F}{Btu}$$

For the air gap, look up Thermal Resistances of Plane Air Spaces in the Reference Handbook and use the table, noting the following details for this situation: (1) The position of the air space is horizontal. (2) The direction of heat flow is up. (3) The mean temperature is $50°F$. The $\Delta T = 20°F$. (4) The temperature differentials listed are 10°F and 30°F; therefore, it will be necessary to interpolate. (5) The effective emittance is a function of the emissivities of the two parallel planes on either side of the air gap, in this case, pinewood and acoustical tile, both of which are $\varepsilon = 0.9$. Search for Parallel Planes in the Heat Transfer / Radiation section of the