43.19 A house has a flat roof comprised of asphalt shingles on top of $2in$ of roof insulation on top of $3in$ of southern pine wood. Below the pinewood there is a $3\frac{1}{2}$ inch air gap followed by $\frac{7}{8}$ inch acoustical ceiling with density $21\frac{lb}{ft^3}$. The pine and the acoustical tiles both have an emissivity of 0.9. The outdoor design temperature is $50°F$ and the indoor design temperature is $70°F$. Assume $15mph$ wind. What is the rate of heat loss per unit area through the roof?

A. $1.5\frac{Btu}{hr \cdot ft^2}$

B. $1.7\frac{Btu}{hr \cdot ft^2}$

C. $1.8\frac{Btu}{hr \cdot ft^2}$

D. $2.0\frac{Btu}{hr \cdot ft^2}$

Calculate the resistance for each layer of the roof and combine to determine the total resistance.

For the outside surface, look up Surface Film Coefficients in the Reference Handbook:

$$R_o = \frac{1}{h_o} = \frac{1}{\left(6.00\frac{Btu}{hr \cdot ft^2 \cdot °F}\right)} = .17\frac{hr \cdot ft^2 \cdot °F}{Btu}$$

For the asphalt shingles, look up Building Materials or Roofing or Asphalt Shingles in the Reference Handbook and select the resistance for the thickness listed:

$$R_{shingles} = .44\frac{hr \cdot ft^2 \cdot °F}{Btu}$$

For the roof insulation, look up Building Materials or Roof Insulation in the Reference Handbook and select the resistance per inch of material thickness. Multiply by the thickness to obtain the resistance for the insulation:

$$R_{insulation} = \left(2.94\frac{hr \cdot ft^2 \cdot °F}{Btu \cdot in}\right)(2in) = 5.88\frac{hr \cdot ft^2 \cdot °F}{Btu}$$

For the southern pine, look up Building Materials or Softwoods or Southern Pine in the Reference Handbook and select the average resistance per inch of material thickness. Multiply by the thickness to obtain the resistance for the pine:

$$R_{pine} = \left(0.95\frac{hr \cdot ft^2 \cdot °F}{Btu \cdot in}\right)(3in) = 2.85\frac{hr \cdot ft^2 \cdot °F}{Btu}$$

For the air gap, look up Thermal Resistances of Plane Air Spaces in the Reference Handbook and use the table, noting the following details for this situation: (1) The position of the air space is horizontal. (2) The direction of heat flow is up. (3) The mean temperature is $50°F$. The $\Delta T = 20°F$. (4) The temperature differentials listed are 10°F and 30°F; therefore, it will be necessary to interpolate. (5) The effective emittance is a function of the emissivities of the two parallel planes on either side of the air gap, in this case, pinewood and acoustical tile, both of which are $\varepsilon = 0.9$. Search for Parallel Planes in the Heat Transfer / Radiation section of the

Reference Handbook and notice the denominator of the first formula. This is an expression for effective emittance as a function of two emissivities of parallel planes:

$$\varepsilon_{eff} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 = \frac{1}{.9} + \frac{1}{.9} - 1 = .82$$

Interpolate:

$$R_{air\,gap} = \frac{.84\frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} + .93\frac{hr \cdot ft^2 \cdot ^\circ F}{Btu}}{2} = .885\frac{hr \cdot ft^2 \cdot ^\circ F}{Btu}$$

For the acoustical tile, look up Building Materials or Insulating Materials or Acoustical Tile in the Reference Handbook and select the entry with the density given, $21\frac{lb}{ft^3}$, and use the average resistance per inch of material thickness. Multiply by the thickness to obtain the resistance for the tile:

$$R_{tile} = \left(2.7\frac{hr \cdot ft^2 \cdot ^\circ F}{Btu \cdot in}\right)\left(\frac{7}{8}in\right) = 2.36\frac{hr \cdot ft^2 \cdot ^\circ F}{Btu}$$

For the inside surface, look up Surface Film Coefficients in the Reference Handbook and select the value for a horizontal roof with the direction of heat flow upward for winter:

$$R_i = \frac{1}{h_i} = \frac{1}{\left(1.63\frac{Btu}{hr \cdot ft^2 \cdot ^\circ F}\right)} = .61\frac{hr \cdot ft^2 \cdot ^\circ F}{Btu}$$

Determine R_{total}:

$$R_{total} = R_o + R_{shingles} + R_{insulation} + R_{pine} + R_{air\,gap} + R_{tile} + R_i$$

$$R_{total} = .17 + .44 + 5.88 + 2.85 + .885 + 2.36 + .61 = 13.2\frac{hr \cdot ft^2 \cdot ^\circ F}{Btu}$$

Determine the overall coefficient of heat transfer, U:

$$U = \frac{1}{R_{total}} = \frac{1}{\left(13.2\frac{hr \cdot ft^2 \cdot ^\circ F}{Btu}\right)} = .076\frac{Btu}{hr \cdot ft^2 \cdot ^\circ F}$$

Write an expression for the heat transfer per unit area based on the combined heat transfer formula:

$$\dot{Q} = UA\Delta T \rightarrow \dot{q} = \frac{\dot{Q}}{A} = U\Delta T$$

$$\dot{q} = \left(.076\frac{Btu}{hr \cdot ft^2 \cdot ^\circ F}\right)(70^\circ F - 50^\circ F) = 1.52\frac{Btu}{hr \cdot ft^2}$$

Answer A