

42.7 48°F chilled water flows in a 200 ft long 8 in steel pipe with 2 in of insulation through a room maintained at 73°F. The insulation has a thermal conductivity of $1.8 \frac{\text{Btu}\cdot\text{in}}{\text{hr}\cdot\text{ft}^2\cdot^\circ\text{F}}$. The overall coefficient of heat transfer for the outside of the insulation to the surrounding room including convection and radiation is $2 \frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2\cdot^\circ\text{F}}$. What is the surface temperature of the outside of the insulation?

- A. 55.8°F
- B. 59.8°F
- C. 61.2°F
- D. 66.3°F

Heat is gained from the ambient space and enters the insulation according to an **overall heat transfer coefficient** for the insulation (which includes both convection and radiation). The same quantity of heat then propagates through the insulation by conduction based on the thickness and thermal conductivity of the insulation. Heat travels through the thickness of the pipe by conduction, and finally, into the flowing water by convection.

Neglect the heat transfer by conduction through the thickness of the pipe because the steel is highly conductive in comparison to the insulation, and no thermal conductivity was given for the pipe.

Neglect the heat transfer by convection into the water as the flow is likely to be turbulent and the chilled water temperature will quickly reach equilibrium with the pipe.

Therefore, consider only the heat gain by the insulation from the room by convection, and the heat gain by conduction through the insulation. These quantities must be equal.

Write an expression for the heat gain by the insulation from the room as though it were a **composite plane wall**:

$$Q_{conv,rad} = U_o A \Delta T = U_o A (T_{room} - T_{surface})$$

Write an expression for the heat gain by conduction through the insulation based on the formula for **conduction through a cylindrical wall**:

$$Q_{cond} = 2\pi L \frac{k}{\ln\left(\frac{r_2}{r_1}\right)} (T_{surface} - T_{water})$$

Set these two expressions equal. Recognize area is the surface area of the outside of the insulation where the conduction and radiation are acting on the insulation, which is given by $2\pi r_2 L$, where $2\pi L$ cancels from both sides leaving r_2 on the left.

$$Q_{conv,rad} = Q_{cond}$$

$$U_o A (T_{room} - T_{surface}) = 2\pi L \frac{k}{\ln\left(\frac{r_2}{r_1}\right)} (T_{surface} - T_{water})$$

$$r_2 U_o (T_{room} - T_{surface}) = \frac{k}{\ln\left(\frac{r_2}{r_1}\right)} (T_{surface} - T_{water})$$

Substitute and solve for $T_{surface}$. Note the thermal conductivity is given per inch, and the denominator takes care of the thickness of the insulation. $r_2 = 6in$ and $r_1 = 4in$ represent the outer and inner radius of the insulation, respectively, for a nominal $8in$ pipe with $2in$ thick insulation:

$$(6in) \left(2 \frac{Btu}{hr \cdot ft^2 \cdot ^\circ F} \right) (73^\circ F - T_{surface}) = \frac{\left(1.8 \frac{Btu \cdot in}{hr \cdot ft^2 \cdot ^\circ F} \right)}{\ln\left(\frac{6in}{4in}\right)} (T_{surface} - 48^\circ F)$$

Solve algebraically and confirm units along the way.

$$T_{surface} = 66.3^\circ F$$

Answer D

42.8 60gpm of hot water is produced by a steam heat exchanger. 5psig saturated steam is provided, and the condensate leaves as a saturated liquid. The flow rate of steam is $2000 \frac{lb_m}{hr}$. If cold water enters at $55^\circ F$, what is the temperature of leaving hot water? Neglect losses.

- A. $113^\circ F$
- B. $116^\circ F$
- C. $119^\circ F$
- D. $122^\circ F$

All of the heat given up by the steam is added to the water, since losses are to be neglected. Set the heat loss for the steam equal to the heat gain for the water. Write an expression for the steam based on change in enthalpy and an expression for the water using the sensible heating rule of thumb for water:

$$\dot{Q}_{steam} = \dot{Q}_{water}$$

$$\dot{m}_{steam} \Delta h = 500gpm \Delta T$$

Use the table **Properties of Saturated Water** (by pressure) to look up the enthalpy of saturated steam and saturated liquid at 5psig:

$$h_f = 196 \frac{Btu}{lb}$$

$$h_g = 1156 \frac{Btu}{lb}$$