

42.16 A sous vide immersion cooker operates by way of a flat 6in square vertical plate on one side of a water bath, maintaining the water temperature for many hours. Moments after being turned on, the plate is 140°F and the water is 60°F. What is the rate of heat transfer? Refer to the table below for properties of water.

T (°F)	ρ (lbm/ft ³)	c_p (Btu/lbm-°F)	μ (lbm/ft-sec)	ν (ft ² /sec)	k (Btu/hr-ft-°F)	Pr	β (1/°F)
32	62.4	1.01	1.20×10^{-3}	1.93×10^{-5}	0.319	13.7	-0.37×10^{-4}
40	62.4	1.00	1.04×10^{-3}	1.67×10^{-5}	0.325	11.6	0.20×10^{-4}
50	62.4	1.00	0.88×10^{-3}	1.40×10^{-5}	0.332	9.55	0.49×10^{-4}
60	62.3	0.999	0.76×10^{-3}	1.22×10^{-5}	0.340	8.03	0.85×10^{-4}
70	62.3	0.998	0.658×10^{-3}	1.06×10^{-5}	0.347	6.82	1.2×10^{-4}
80	62.2	0.998	0.578×10^{-3}	0.93×10^{-5}	0.353	5.89	1.5×10^{-4}
90	62.1	0.997	0.514×10^{-3}	0.825×10^{-5}	0.359	5.13	1.8×10^{-4}
100	62.0	0.998	0.458×10^{-3}	0.740×10^{-5}	0.364	4.52	2.0×10^{-4}
150	61.2	1.00	0.292×10^{-3}	0.477×10^{-5}	0.384	2.74	3.1×10^{-4}
200	60.1	1.00	0.205×10^{-3}	0.341×10^{-5}	0.394	1.88	4.0×10^{-4}

- A. $2540 \frac{Btu}{hr}$
- B. $3180 \frac{Btu}{hr}$
- C. $4620 \frac{Btu}{hr}$
- D. $5000 \frac{Btu}{hr}$

For convection, apply **Newton's Law of Cooling**:

$$\dot{Q} = hA\Delta T$$

For **Natural (Free) Convection** involving a **Vertical Flat Plate in Large Body of Stationary Fluid**, the convection heat transfer coefficient is:

$$\bar{h} = C \left(\frac{k}{L} \right) Ra_L^n$$

where C and n are constants, k is the thermal conductivity, L is the vertical height of the plate, and Ra_L is the **Rayleigh Number**, which can be calculated using:

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} Pr$$

where g is acceleration due to gravity, β is the coefficient of thermal expansion for water, T_s is the average surface temperature of the plate, T_∞ is the bulk (ambient) temperature of the water, L is the vertical height of the plate, ν is the kinematic viscosity, and Pr is the Prandtl number. Use the table **Properties of Water** and the table provided to obtain Pr , ν , β , and k , then calculate Ra_L .

Calculate the film temperature for looking up various parameters required to determine the Rayleigh Number. Note that the film temperature dictates the values of most parameters in the Rayleigh Number formula. Looking the parameters up at the surface temperature or bulk temperature will lead to errors.

$$T_f = \frac{T_s + T_\infty}{2} = \frac{140^\circ F + 60^\circ F}{2} = 100^\circ F$$

$$Ra_L = \frac{(32.2 \frac{ft}{s^2}) \left(\frac{2 \times 10^{-4}}{^\circ F} \right) (140^\circ F - 60^\circ F) (0.5 ft)^3 (4.52)}{\left(0.74 \times 10^{-5} \frac{ft^2}{s} \right)^2} = 5.32 \times 10^9$$

Based on the range of Ra_L select the appropriate constants C and n :

$$10^9 < Ra_L < 10^{13}$$

$$C = 0.10$$

$$n = 1/3$$

Calculate \bar{h} :

$$\bar{h} = C \left(\frac{k}{L} \right) Ra_L^n = (.1) \left(\frac{\left(0.364 \frac{Btu}{hr \cdot ft^2 \cdot ^\circ F} \right)}{(.5 ft)} \right) (5.32 \times 10^9)^{\frac{1}{3}} = 127 \frac{Btu}{hr \cdot ft^2 \cdot ^\circ F}$$

Determine the rate of heat transfer, \dot{Q} :

$$\dot{Q} = hA\Delta T = \left(127 \frac{Btu}{hr \cdot ft^2 \cdot ^\circ F} \right) (0.5 ft)^2 (140^\circ F - 60^\circ F) = 2541 \frac{Btu}{hr}$$

Answer A

42.17 Cold water initially at $55^\circ F$ is heated to $120^\circ F$ to make potable hot water using a parallel flow shell and tube heat exchanger. Low temperature hot water from a boiler enters the heat exchanger at $160^\circ F$ and leaves at $130^\circ F$. What is the log mean temperature difference?

- A. $40^\circ F$
- B. $45^\circ F$
- C. $56^\circ F$
- D. $58^\circ F$

In a **parallel flow** heat exchanger, the temperature of the cold and hot stream will approach but never reach one another. The smallest temperature differential between the two streams will be observed at the outlet and the highest temperature differential will be observed at the inlet.

Call the inlet temperature differential ΔT_A and the outlet temperature differential ΔT_B :

$$\Delta T_A = 160^\circ F - 55^\circ F = 105^\circ F$$