

$$\Delta T_B = 130^\circ F - 120^\circ F = 10^\circ F$$

Recall the simplified **LMTD** equation offered in solution 10.2. Substitute and solve for the log mean temperature difference:

$$\Delta T_{lm} = \frac{\Delta T_A - \Delta T_B}{\ln\left(\frac{\Delta T_A}{\Delta T_B}\right)} = \frac{105^\circ F - 10^\circ F}{\ln\left(\frac{105^\circ F}{10^\circ F}\right)} = 40.4^\circ F$$

Answer A

42.18 In the summer, a warehouse is maintained at $72^\circ F$ by several chilled beams supplied with $58^\circ F$ chilled water which returns at $65^\circ F$. Treating the chilled beam coil as a heat exchanger, what is the log mean temperature difference?

- A. $7^\circ F$
- B. $10^\circ F$
- C. $11^\circ F$
- D. $14^\circ F$

To treat the chilled beam coil as a heat exchanger, consider the chilled water to be the cold stream, entering at $58^\circ F$ and leaving at $65^\circ F$. The air side will represent the warm stream, however, we do not know the supply and return temperature. We could assume the return temperature is representative of the room and guess that the supply air temperature is somewhat lower. This would not be unreasonable, although the supply air temperature chosen would be an outright guess.

Thinking about the way a ceiling mounted chilled beam works, in particular the *passive* style, the stratification of the air naturally brings warm air into contact with the chilled beam, which cools the air which gently falls away, promoting mixing vertically throughout the space without the use of a fan. Also since the chilled water is supplied at a higher temperature, there is an opportunity to capture savings not only by omitting the fan, but also by raising the chilled water setpoint for the system.

As an approximation, suppose the passive chilled beam is holding the space in equilibrium, and the supply and return temperatures are nearly equal. Note this is *not strictly* true,

$$SAT \approx RAT = 72^\circ F$$

To find the **LMTD**, calculate ΔT_A at the inlet and ΔT_B at the outlet:

$$\Delta T_A = 72^\circ F - 58^\circ F = 14^\circ F$$

$$\Delta T_B = 72^\circ F - 65^\circ F = 7^\circ F$$

Substitute and solve for the log mean temperature difference:

$$\Delta T_{lm} = \frac{\Delta T_A - \Delta T_B}{\ln\left(\frac{\Delta T_A}{\Delta T_B}\right)} = \frac{14^\circ F - 7^\circ F}{\ln\left(\frac{14^\circ F}{7^\circ F}\right)} = 10.1^\circ F$$

Answer B

42.19 An $18in \times 24in$ uninsulated duct runs through a $100ft$ auditorium. Air in the duct has an initial temperature of $55^\circ F$ and a velocity of $800ft$ per min. The auditorium is maintained at $75^\circ F$ by a dedicated system. The outside film coefficient for the duct is $1.4 \frac{Btu}{hr \cdot ft^2 \cdot ^\circ F}$. What is the temperature of the air in the duct as it exits the room?

- A. $57^\circ F$
- B. $58^\circ F$
- C. $59^\circ F$
- D. $60^\circ F$

This is a convection problem involving heat gain into a duct from a room held at constant temperature. The duct is not circular but the **hydraulic diameter** may be determined such that circular duct formulas may be used:

$$D_h = \frac{4A}{P} = \frac{4(1.5ft)(2ft)}{[(2)(2ft) + (2)(1.5ft)]} = \frac{12ft^2}{7ft} = 1.71ft$$

Next find the **Reynolds Number** to determine whether the flow is laminar or turbulent. Use the table **Properties of Air at Atmospheric Pressure** and choose the temperature of $60^\circ F$ for convenience.

$$Re = \frac{vD}{\nu} = \frac{\left(800 \frac{ft}{min}\right) \left(\frac{1min}{60sec}\right) (1.71ft)}{\left(15.8 \times 10^{-5} \frac{ft^2}{s}\right)} \approx 144,300 > 10,000 \text{ (turbulent)}$$

Having established that the flow is turbulent and that the duct may be treated as round, use the **Sieder-Tate equation** found in the Reference Handbook by searching **Turbulent Flow in Circular Tubes**. This equation specifies the **Nusselt Number**, Nu , which will be useful for determining the convection heat transfer coefficient. Use the table **Properties of Air at Low Pressure** to look up the value for the Prantl Number, Pr . Since the air temperatures inside and outside the duct are not dramatically different, assume the dynamic viscosity of air at the bulk temperature, μ_b , is approximately equal to the dynamic viscosity of air at the inside surface temperature of the duct, μ_s . Calculate Nu :

$$Nu_D = 0.023 Re_D^{0.8} Pr^{1/3} \left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$\mu_b \approx \mu_s \rightarrow \frac{\mu_b}{\mu_s} \approx 1 \rightarrow Nu_D \approx 0.023 Re_D^{0.8} Pr^{1/3}$$