$$\Delta T_{lm} = \frac{\Delta T_A - \Delta T_B}{ln\left(\frac{\Delta T_A}{\Delta T_B}\right)} = \frac{14°F - 7°F}{ln\left(\frac{14°F}{7°F}\right)} = 10.1°F$$

Answer B

42.19 An $18in \times 24in$ uninsulated duct runs through a $100ft$ auditorium. Air in the duct has an initial temperature of $55°F$ and a velocity of $800ft$ per min. The auditorium is maintained at $75°F$ by a dedicated system. The outside film coefficient for the duct is $1.4\frac{Btu}{hr \cdot ft^2 \cdot °F}$. What is the temperature of the air in the duct as it exits the room?

A. $57°F$

B. $58°F$

C. $59°F$

D. $60°F$

This is a convection problem involving heat gain into a duct from a room held at constant temperature. The duct is not circular but the hydraulic diameter may be determined such that circular duct formulas may be used:

$$D_h = \frac{4A}{P} = \frac{4\,(1.5ft)\,(2ft)}{[(2)\,(2ft) + (2)\,(1.5ft)]} = \frac{12ft^2}{7ft} = 1.71ft$$

Next find the Reynolds Number to determine whether the flow is laminar or turbulent. Use the table Properties of Air at Atmospheric Pressure and choose the temperature of 60°F for convenience.

$$Re = \frac{vD}{\nu} = \frac{\left(800\frac{ft}{min}\right)\left(\frac{1min}{60sec}\right)(1.71ft)}{\left(15.8 \times 10^{-5}\frac{ft^2}{s}\right)} \approx 144,300 > 10,000\ (turbulent)$$

Having established that the flow is turbulent and that the duct may be treated as round, use the Sieder-Tate equation found in the Reference Handbook by searching Turbulent Flow in Circular Tubes. This equation specifies the Nusselt Number, Nu, which will be useful for determining the convection heat transfer coefficient. Use the table Properties of Air at Low Pressure to look up the value for the Prantl Number, Pr. Since the air temperatures inside and outside the duct are not dramatically different, assume the dynamic viscosity of air at the bulk temperature, μ_b, is approximately equal to the dynamic viscosity of air at the inside surface temperature of the duct, μ_s. Calculate Nu:

$$Nu_D = 0.023Re_D^{0.8}Pr^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$\mu_b \approx \mu_s \rightarrow \frac{\mu_b}{\mu_s} \approx 1 \rightarrow Nu_D \approx 0.023Re_D^{0.8}Pr^{1/3}$$

$$Nu_D = (0.023)\,(144,300)^{0.8}\,(.715)^{1/3} = 275.8$$

A formula that is offered in a few different scenarios in the Reference Handbook but never stated generally is the equation for the Nusselt Number and how it can be related to the convection heat transfer coefficient. It comes up periodically and is probably worth memorizing, ideally in the context of solving actual heat transfer problems such as this one. Rearrange to solve for h_i, the *inside* film coefficient. Look up the thermal conductivity for air in the table Properties of Air at Low Pressure. Note the *outside* film coefficient was given, and both will be needed to determine the total resistance across the thickens of the duct.

$$Nu = \frac{hD}{k} \rightarrow h_i = \frac{(Nu)\,(k)}{D} = \frac{(275.8)\left(.0145\frac{Btu}{hr\cdot ft\cdot {}^\circ F}\right)}{1.71ft} = 2.34\frac{Btu}{hr\cdot ft^2\cdot {}^\circ F}$$

Find the total resistance, R_T:

$$R_T = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{2.34\frac{Btu}{hr\cdot ft^2\cdot {}^\circ F}} + \frac{1}{1.4\frac{Btu}{hr\cdot ft^2\cdot {}^\circ F}} = 1.14\frac{hr\cdot ft^2\cdot {}^\circ F}{Btu}$$

Find the overall coefficient of heat transfer, U:

$$U = \frac{1}{R_T} = \frac{1}{1.14\frac{hr\cdot ft^2\cdot {}^\circ F}{Btu}} = .876\frac{Btu}{hr\cdot ft^2\cdot {}^\circ F}$$

The heat entering the duct by convection heats the air inside the duct as it passes through the room. Write an equation for the heat transfer into the duct by convection, $\dot{Q}_t$ (as governed by the overall coefficient of heat transfer just determined) and the sensible heat gain by the air, $\dot{Q}_s$, based on the sensible heating rule of thumb for air.

$$\dot{Q}_t = \dot{Q}_s$$

$$UA\Delta T_1 = 1.08cfm\Delta T_2$$

Clarify the temperature differentials for either side of the equation, as they are not the same. For the heat gain, it is the ambient temperature outside the duct as compared to the bulk temperature inside the duct, which is the average of the entering and exiting air.

$$\Delta T_1 = 75^\circ F - T_b$$

where:

$$T_b = \frac{T_{exit} + 55^\circ F}{2} = \frac{T_{exit}}{2} + 27.5^\circ F$$

Substituting:

$$\Delta T_1 = 75^\circ F - \left[\frac{T_{exit}}{2} + 27.5^\circ F\right] = 47.5^\circ F - \frac{T_{exit}}{2}$$

For the air inside the duct, the temperature differential is more straightforward, simply the exit temperature minus the entering temperature:

$$\Delta T_2 = T_{exit} - 55°F$$

Multiply area and velocity to get the volume flow rate of air in the duct:

$$Q_{[cfm]} = VA = \left(800\frac{ft}{min}\right)\left(3ft^2\right) = 2400cfm$$

Return to previous equation and solve:

$$UA\Delta T_1 = 1.08cfm\Delta T_2$$

$$\left(.876\frac{Btu}{hr \cdot ft^2 \cdot °F}\right)\left(700ft^2\right)\left(47.5°F - \frac{T_{exit}}{2}\right) = (1.08)\,(2400)\,(T_{exit} - 55)$$

Since the expression on the right side is a "rule of thumb" and does not normally require including units, trust the units have worked out thus far and both sides will ultimately simplify to $\frac{Btu}{hr}$. For expedience, solve as a pure algebra equation with one unknown, T_{exit}:

$$613.2\left(47.5°F - \frac{T_{exit}}{2}\right) = 2592\,(T_{exit} - 55)$$

$$29,127 - 306.6\,(T_{exit}) = 2592\,(T_{exit}) - 142,560$$

$$171,687 = 2898.6\,(T_{exit})$$

$$T_{exit} = 59.2°F$$

Answer C