42.20 A clean, new domestic hot water heater uses $25psia$ saturated steam to heat $50gpm$ of water from $60°F$ to $130°F$. After a year of operation, the overall fouling factor is found to be $0.001\frac{hr \cdot ft^2 \cdot °F}{Btu}$. The total heat exchange surface area is $20ft^2$. What will the hot water supply (outlet) temperature be for the same $50gpm$ volume flow rate entering at $60°F$ under fouled conditions?

A. $100°F$

B. $110°F$

C. $120°F$

D. $125°F$

The formula needed to analyze the change in performance due to fouling can be derived from the generalized formula presented in the Reference Handbook under Overall Heat-Transfer Coefficient for Concentric Tube and Shell-and-Tube Heat Exchangers by applying a few simplifying assumptions.

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{ln\left(\frac{D_o}{D_i}\right)}{2\pi kL} + \frac{R_{fo}}{A_o} + \frac{1}{h_o A_o}$$

Assume the internal and external area of the heat exchanger are approximately the same i.e. $A \approx A_i \approx A_o \approx 2\pi rL$. Therefore all terms can be multiplied by area as such:

$$\frac{1}{U} = \frac{1}{h_i} + R_{fi} + \frac{r \cdot ln\left(\frac{D_o}{D_i}\right)}{k} + R_{fo} + \frac{1}{h_o}$$

Now collapse the internal and external fouling factors, R_{fi} and R_{fo}, respectively, into a single parameter inclusive of losses on both sides i.e. $R_f = R_{fi} + R_{fo}$. Also assume that k, the thermal conductivity of the heat exchanger is relatively large such that the middle term becomes negligible. Then:

$$\frac{1}{U_{fouled}} = \frac{1}{h_i} + R_f + \frac{1}{h_o}$$

The formula above would be applicable when the heat exchanger is fouled i.e. the fouling factor is nonzero: $R_f > 0$

When the heat exchanger is clean, the fouling factor has a value of zero and need not be included: $R_f = 0$

$$\frac{1}{U_{clean}} = \frac{1}{h_i} + \frac{1}{h_o}$$

By substitution:

$$\frac{1}{U_{fouled}} = \frac{1}{U_{clean}} + R_f$$

$$R_f = \frac{1}{U_{fouled}} - \frac{1}{U_{clean}}$$

This formula is readily available in the Mechanical Engineering Reference Manual; however, it is not given in the NCEES Reference Handbook. It may be advisable to memorize it in order to skip the derivation above (or at least be aware of the opportunity to simplify when necessary).

Use the steam table Properties of Saturated Water (by Pressure) to look up the temperature of the steam used for heating. Note the steam is condensing at constant temperature, therefore the hot side of the heat exchanger has the same entering and leaving temperature (assuming the steam leaves as a saturated liquid or saturated mixture and does not become sub-cooled).

$$T_{sat@25psia} = 240°F$$

Find the log mean temperature difference, LMTD, for the heat exchanger by calculating ΔT_A at the inlet and ΔT_B at the outlet:

$$\Delta T_A = 240°F - 60°F = 180°F$$

$$\Delta T_B = 240°F - 130°F = 110°F$$

Substitute and solve for the log mean temperature difference:

$$\Delta T_{lm} = \frac{\Delta T_A - \Delta T_B}{ln\left(\frac{\Delta T_A}{\Delta T_B}\right)} = \frac{180°F - 110°F}{ln\left(\frac{180°F}{110°F}\right)} = 142.1°F$$

Find the overall coefficient of heat transfer for the heat exchanger when clean, U_{clean}, by equating the heat added by the heat exchanger to the heat gained by the water using the sensible heating rule of thumb for water. Note the right side of the equation ultimately has units of $\frac{Btu}{hr}$.

$$\dot{Q}_{clean} = \dot{Q}_{water}$$

$$U_{clean} A \Delta T_{lm} = 500 gpm \Delta T_w = (500)(50)(130 - 60) = 1.75 \times 10^6 \frac{Btu}{hr}$$

$$U_{clean} = \frac{\left(1.75 \times 10^6 \frac{Btu}{hr}\right)}{(20 ft^2)(142.1°F)} = 616 \frac{Btu}{hr \cdot ft^2 \cdot °F}$$

Calculate U_{fouled} by applying the overall fouling factor given, R_f:

$$\frac{1}{U_{fouled}} = \frac{1}{U_{clean}} + R_f = \frac{1}{616 \frac{Btu}{hr \cdot ft^2 \cdot °F}} + .001 \frac{hr \cdot ft^2 \cdot °F}{Btu} = .00262 \frac{hr \cdot ft^2 \cdot °F}{Btu}$$

$$U_{fouled} = \frac{1}{.00262 \frac{hr \cdot ft^2 \cdot °F}{Btu}} = 381 \frac{Btu}{hr \cdot ft^2 \cdot °F}$$

Applying the overall coefficient of heat transfer for the heat exchanger when fouled, find the total heat transfer, $\dot{Q}_{fouled}$. Use the same log mean temperature difference, but note that iteration may be necessary if the temperature changes significantly.

$$\dot{Q}_{fouled} = U_{fouled} A \Delta T_{lm}$$

$$\dot{Q}_{fouled} = \left(381 \frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}\right) \left(20 ft^2\right) \left(142.1^\circ F\right) = 1.083 \times 10^6 \frac{Btu}{hr}$$

Find the new water ΔT considering the reduction in heat transfer under fouled conditions:

$$\dot{Q}_{water} = 500 gpm \Delta T_w \rightarrow \Delta T_w = \frac{\dot{Q}_{water}}{500 gpm}$$

$$\Delta T_w = \frac{1.083 \times 10^6}{(500)(50)} = 43.3^\circ F$$

$$\Delta T_w = T_2 - T_1 = T_2 - 60^\circ F = 43.3^\circ F \rightarrow T_2 = 103.3^\circ F$$

Although it may be tempting to select Answer choice A at this point, the assumption that the log mean temperature difference is $142.1^\circ F$ must be challenged. Re-calculate the LMTD under fouled conditions:

$$\Delta T_A = 240^\circ F - 60^\circ F = 180^\circ F$$

$$\Delta T_B = 240^\circ F - 103.3^\circ F = 136.7^\circ F$$

$$\Delta T_{lm} = \frac{\Delta T_A - \Delta T_B}{ln\left(\frac{\Delta T_A}{\Delta T_B}\right)} = \frac{180^\circ F - 136.7^\circ F}{ln\left(\frac{180^\circ F}{136.7^\circ F}\right)} = 157.4^\circ F$$

Recalculate $\dot{Q}_{fouled}$:

$$\dot{Q}_{fouled} = \left(381 \frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}\right) \left(20 ft^2\right) \left(157.4^\circ F\right) = 1.2 \times 10^6 \frac{Btu}{hr}$$

Recalculate the water ΔT:

$$\Delta T_w = \frac{1.2 \times 10^6}{(500)(50)} = 48^\circ F$$

Recalculate the leaving water temperature, T_2:

$$\Delta T_w = T_2 - T_1 = T_2 - 60^\circ F = 48^\circ F \rightarrow T_2 = 108^\circ F$$

Note further iteration will result in a reduction in the outlet temperature, but the amount of change between iterations decreases as the solution converges, and it will ultimately land closer to $110^\circ F$ than $100^\circ F$.

Due to time constraints, the PE exam is not likely to pose questions which require an iterative solution; however this example has been included for your awareness of the possibility.

Answer B