$$\dot{Q}_r = (.9)\left(.1713 \times 10^{-8} \frac{Btu}{hr \cdot ft^2 \cdot {}^\circ R}\right)(73.3ft^2)\left[(610^\circ R)^4 - (526^\circ R)^4\right]$$

$$\dot{Q}_r = 7000 \frac{Btu}{hr}$$

Add the convection and radiation to determine the total heat transfer:

$$\dot{Q}_t = \dot{Q}_c + \dot{Q}_r = 4700 \frac{Btu}{hr} + 7000 \frac{Btu}{hr} = 11,700 \frac{Btu}{hr}$$

Answer D

42.22 **$400cfm$ of $50^\circ F$ conditioned supply air enters a $10in$ round uninsulated duct with an outside surface emissivity of 0.4 and an average surface temperature of $58^\circ F$. The duct travels through a $30ft$ room in which the ambient air and all contents and surfaces are maintained at $76^\circ F$. What is the temperature of the air exiting the room? Refer to the table below for properties of air.**

T (°F)	ρ (lbm/ft^3)	c_p (Btu/lbm-°F)	μ (lbm/ft-sec)	ν (ft^2/sec)	k (Btu/hr-ft-°F)	Pr	β (1/°F)	$\frac{g\beta\rho^2}{\mu^2}$ (1/ft^3-°F)
0	0.086	0.239	1.110×10^{-5}	0.130×10^{-3}	0.0133	0.73	2.18×10^{-3}	4.2×10^{6}
32	0.081	0.240	1.165×10^{-5}	0.145×10^{-3}	0.0140	0.72	2.03×10^{-3}	3.16×10^{6}
100	0.071	0.240	1.285×10^{-5}	0.180×10^{-3}	0.0154	0.72	1.79×10^{-3}	1.76×10^{6}
200	0.060	0.241	1.440×10^{-5}	0.239×10^{-3}	0.0174	0.72	1.52×10^{-3}	0.850×10^{6}
300	0.052	0.243	1.610×10^{-5}	0.306×10^{-3}	0.0193	0.71	1.32×10^{-3}	0.444×10^{6}
400	0.046	0.245	1.75×10^{-5}	0.378×10^{-3}	0.0212	0.689	1.16×10^{-3}	0.258×10^{6}
500	0.0412	0.247	1.890×10^{-5}	0.455×10^{-3}	0.0231	0.683	1.04×10^{-3}	0.159×10^{6}
600	0.0373	0.250	2.000×10^{-5}	0.540×10^{-3}	0.0250	0.685	0.943×10^{-3}	0.106×10^{6}
700	0.0341	0.253	2.14×10^{-5}	0.625×10^{-3}	0.0268	0.690	0.862×10^{-3}	70.4×10^{3}
800	0.0314	0.256	2.25×10^{-5}	0.717×10^{-3}	0.0286	0.697	0.794×10^{-3}	49.8×10^{3}
900	0.0291	0.259	2.36×10^{-5}	0.815×10^{-3}	0.0303	0.705	0.735×10^{-3}	36.0×10^{3}
1000	0.0271	0.262	2.47×10^{-5}	0.917×10^{-3}	0.0319	0.713	0.685×10^{-3}	26.5×10^{3}
1500	0.0202	0.276	3.00×10^{-5}	1.47×10^{-3}	0.0400	0.739	0.510×10^{-3}	7.45×10^{3}
2000	0.0161	0.286	3.54×10^{-5}	2.14×10^{-3}	0.0471	0.753	0.406×10^{-3}	2.84×10^{3}
2500	0.0133	0.292	3.69×10^{-5}	2.80×10^{-3}	0.051	0.763	0.338×10^{-3}	1.41×10^{3}
3000	0.0114	0.297	3.85×10^{-5}	3.39×10^{-3}	0.054	0.765	0.289×10^{-3}	0.815×10^{3}

A. $51^\circ F$

B. $52^\circ F$

C. $53^\circ F$

D. $54^\circ F$

Heat is gained by the air in the duct by forced convection. This is the primary mode of heat transfer internal to the duct.

Heat is gained by the duct via both natural convection and radiation. This is driven by the conditions external to the duct.

Through the duct itself, there is convection across the thickness of the sheet metal. Since the duct is uninsulated and the thermal conductivity of sheet metal is large, neglect conduction and equate the internal heat gain to the external heat gain.

$$\dot{Q}_{conv,int} = \dot{Q}_{ext} = \dot{Q}_{conv,ext} + \dot{Q}_{rad,ext}$$

Start by considering the internal heat gain by forced convection. Look up Turbulent Flow in Circular Tubes in the Reference Handbook and refer to the formula below. Assume the dynamic viscosity is approximately equal for the bulk and surface conditions since the temperature difference between the duct surface and the bulk air flow is not significant.

$$Nu_D = 0.023 Re_D^{0.8} Pr^{1/3} \left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$\mu_b \approx \mu_s \rightarrow \frac{\mu_b}{\mu_s} \approx 1 \rightarrow Nu_D \approx 0.023 Re_D^{0.8} Pr^{1/3}$$

Calculate the velocity:

$$v = \frac{Q}{A} = \frac{\left(400 \frac{ft^3}{min}\right)\left(\frac{1min}{60s}\right)}{\left(\frac{\pi}{4}\right)\left(\frac{10}{12} ft\right)^2} = 12.2 \frac{ft}{s}$$

Calculate the Reynolds Number, Re. Use the table Properties of Atmospheric Air for the kinematic viscosity.

$$Re = \frac{vD}{\nu} = \frac{\left(12.2 \frac{ft}{s}\right)\left(\frac{10}{12} ft\right)}{\left(15.5 \times 10^{-5} \frac{ft^2}{s}\right)} = 65,000 > 10,000\ (turbulent)$$

Calculate Nu and use to specify the internal convection heat transfer coefficient, h_i:

$$Nu_D = (0.023)(65,700)^{0.8}(.72)^{1/3} = 147$$

$$Nu = \frac{hD}{k} \rightarrow h_i = \frac{Nu \cdot k}{D}$$

$$h_i = \frac{(147)\left(.0145 \frac{Btu}{hr \cdot ft \cdot {}^\circ F}\right)}{\left(\frac{10}{12} ft\right)} = 2.557 \frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}$$

Write an expression for the internal heat gain by the air flowing in the duct via forced convection. Note the ΔT in the formula is the difference between the *average surface temperature of the duct* and the *average i.e. "bulk" temperature of the air.* Later the bulk temperature of the air will be related to the entering and leaving air temperatures.

$$\dot{Q}_{conv,int} = h_i A \Delta T = \left(2.557 \frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}\right)\left(\pi \left(\frac{10}{12} ft\right)(30 ft)\right)(58^\circ F - T_b)$$

Consider the external heat gain, starting with natural convection. Use Reference Handbook section Long Horizontal Cylinder in Large Body of Stationary Fluid to find the external convection heat transfer coefficient, h_o:

$$h_o = C\left(\frac{k}{D}\right) Ra_D^n$$

where Ra:

$$Ra_D = \frac{g\beta\left(T_s - T_\infty\right)D^3}{\nu^2}Pr$$

Use the table given to select the values for properties of atmospheric air at the film temperature and calculate Ra.

$$T_{film} = \frac{T_s + T_\infty}{2} = \frac{58^\circ F + 76^\circ F}{2} = 67^\circ F$$

$$Ra_D = \frac{\left(32.2\frac{ft}{s^2}\right)\left(1.9\times10^{-3}\frac{1}{^\circ F}\right)\left(76^\circ F - 58^\circ F\right)\left(\frac{10}{12}ft\right)^3(.72)}{\left(16\times10^{-5}\frac{ft^2}{s}\right)^2} = 1.8\times10^7$$

Based on the range of Ra, specify constants C and n:

$$10^7 < Ra < 10^{12}$$

$$C = .125$$

$$n = .333$$

Calculate the convection heat transfer coefficient:

$$h_o = (.125)\left(\frac{.0147\frac{Btu}{hr\cdot ft\cdot ^\circ F}}{\frac{10}{12}ft}\right)\left(1.8\times10^7\right)^{.333} = .574\frac{Btu}{hr\cdot ft^2\cdot ^\circ F}$$

Determine the heat transfer by natural convection. Note the ΔT is the difference between the *surrounding* air temperature and the *surface* temperature of the duct.

$$\dot{Q}_{conv,ext} = h_o A\Delta T = \left(.574\frac{Btu}{hr\cdot ft^2\cdot ^\circ F}\right)\left(\pi\left(\frac{10}{12}ft\right)(30ft)\right)(76^\circ F - 58^\circ F) = 811\frac{Btu}{hr}$$

Consider the external heat gain by radiation. Look up Net Energy Exchange by Radiation Between Two Bodies and use equation:

$$\dot{Q}_r = \varepsilon\sigma A\left(T_1^4 - T_2^4\right)$$

where ε is the emissivity, σ is the Stefan-Boltzmann constant, A is surface area, and the temperatures are the surface temperatures of the duct and the surrounding walls. Note the temperatures must be in absolute terms i.e. degrees Rankine.

Note the emissivity was given.

$$\varepsilon = .4$$

Change the temperatures to Rankine:

$$T_1 = T_{walls} = 76^\circ F + 460^\circ = 536^\circ R$$

$$T_2 = T_{duct} = 58°F + 460° = 518°R$$

Calculate the heat transfer by radiation:

$$\dot{Q}_{rad,ext} = (.4)\left(.1713 \times 10^{-8} \frac{Btu}{hr \cdot ft^2 \cdot °R}\right)\left(\pi\left(\frac{10}{12}ft\right)(30ft)\right)\left[(536°R)^4 - (518°R)^4\right]$$

$$\dot{Q}_{rad,ext} = 567\frac{Btu}{hr}$$

Calculate the total external heat gain:

$$\dot{Q}_{ext} = \dot{Q}_{conv,ext} + \dot{Q}_{rad,ext} = 811\frac{Btu}{hr} + 567\frac{Btu}{hr} = 1378\frac{Btu}{hr}$$

Since the internal heat gain is equal to the external heat gain, set this equal to the internal forced convection expression obtained earlier:

$$\dot{Q}_{conv,int} = \left(2.557\frac{Btu}{hr \cdot ft^2 \cdot °F}\right)\left(\pi\left(\frac{10}{12}ft\right)(30ft)\right)(58°F - T_b) = 1378\frac{Btu}{hr}$$

Simplify:

$$\left(200.8\frac{Btu}{hr \cdot °F}\right)(58°F - T_b) = 1378\frac{Btu}{hr}$$

$$58°F - T_b = \frac{1378\frac{Btu}{hr}}{200.8\frac{Btu}{hr \cdot °F}} = 6.9°F$$

$$T_b = 58°F - 6.9°F = 51.1°F$$

Recall T_b is the *average "bulk"* temperature of the air in the duct which is the average of the entering and leaving air temperatures. The entering air temperature is given. The exiting air temperature is unknown.

$$T_b = \frac{T_{enter} + T_{exit}}{2} = \frac{50°F + T_{exit}}{2} = 51.1°F$$

$$50°F + T_{exit} = 102.2°F$$

$$T_{exit} = 52.2°F$$

Answer B