

46.9 40gpm of water flows through a parallel piping network with two branches, A and B. Branch A consists of 100ft of straight Schedule 40 2in steel pipe. Branch B consists of 60ft of Schedule 40 2in steel pipe, several valves with a total equivalent length of 35ft, and six 90-degree elbows. What is the flow rate through branch A?

- A. 19gpm
- B. 21gpm
- C. 23gpm
- D. 25gpm

Sketch the piping network. Since there are two branches in parallel, the head loss through each branch must be equal. Write the **Darcy** equation for both sides and set them equal.

$$h_{f,A} = h_{f,B}$$

$$\frac{fL_A v_A^2}{2D_A g} = \frac{fL_B v_B^2}{2D_B g}$$

Assume the friction factor is approximately the same for both branches and will cancel out, along with the constants, 2 and g . Also, $D_A = D_B$, so the diameters will cancel out.

$$L_A v_A^2 = L_B v_B^2$$

Rearrange the relation to express the ratio of the velocities through the two branches as a function of the equivalent lengths of the branches.

$$\frac{v_B}{v_A} = \sqrt{\frac{L_A}{L_B}}$$

Recognize from the **Continuity Equation**, $Q = Av$, since the diameters are the same, the areas of the two branches are also the same, therefore the ratio of the velocities is equal to the ratio of the volume flow rates.

$$Q_A = A_A v_A \rightarrow v_A = \frac{Q_A}{A_A}$$

$$Q_B = A_B v_B \rightarrow v_B = \frac{Q_B}{A_B}$$

$$A_A = A_B$$

$$\frac{v_B}{v_A} = \frac{\left(\frac{Q_B}{A_B}\right)}{\left(\frac{Q_A}{A_A}\right)} = \frac{Q_B}{Q_A} = \sqrt{\frac{L_A}{L_B}}$$

The total volume flow rate is the sum of both branches and is given as 40gpm.

$$Q_A + Q_B = 40\text{gpm}$$

If both equivalent lengths, L_A and L_B , were known, it would be possible to solve the system of two equations with two unknowns, Q_A and Q_B , to specify Q_B . However, only L_A is known. $L_A = 100\text{ft}$. The length of branch B is a function of the linear footage, the equivalent length of valves, and the equivalent length of elbows. The latter two are known, however the equivalent length of the elbows is a function of both the diameter and the velocity. Since the velocity is unknown, there is a circular dependency making a direct solution impossible.

As a workaround, refer to the table [Equivalent Lengths for Elbows](#) and note that for a typical range of velocities, for instance $2 - 5\text{fps}$, the equivalent length per elbow for a 2in diameter ranges from $5.1 - 5.9\text{ft}$. Since this will only make up about a quarter of the total length of branch B regardless of where in that range, it is reasonable to guess the volume flow rate through branch B, find the corresponding velocity through branch B and the equivalent length of the elbows, and then use the system of equations to confirm the guess, or recalibrate and perform a second iteration.

Looking at the answer choices, it is clear that more than half of the flow is going to flow through branch A because branch B has a greater total equivalent length (i.e. more resistance, even if the velocity through branch B is fairly low this remains true.) For convenience, select a volume flow of 22gpm for branch A which is intentionally in between choices B and C, and available in the [Steel Pipe Friction Tables](#) to save time calculating the velocity.

$$Q_A = 22\text{gpm}$$

$$Q_B = 40\text{gpm} - 22\text{gpm} = 18\text{gpm}$$

$$v_B = 1.72\text{fps}$$

$$L_{\text{elbow}} \approx 5\text{ft}$$

$$L_B = 60\text{ft} + 35\text{ft} + (6)(5\text{ft}) = 125\text{ft}$$

$$\frac{Q_B}{Q_A} = \sqrt{\frac{L_A}{L_B}} = \sqrt{\frac{100\text{ft}}{125\text{ft}}} = 0.8944$$

$$\frac{Q_B}{Q_A} = \frac{18\text{gpm}}{22\text{gpm}} = 0.8182$$

To close the gap between the ratio $\frac{Q_B}{Q_A}$ computed using both methods, it would be necessary to try a larger value for Q_B and therefore a smaller value for Q_A . Since only one answer choice is smaller, select $Q_A = 21\text{gpm}$.

If time allows, perform a second iteration for full reassurance.

$$Q_A = 21\text{gpm}$$

$$Q_B = 40\text{gpm} - 21\text{gpm} = 19\text{gpm}$$