47.42 An exterior wall is constructed from $\frac{5}{8}in$ plaster board $\left(R = 0.56\frac{hr \cdot ft^2 \cdot ^\circ F}{Btu}\right)$, $3.5in$ batt insulation ($R = 13\frac{hr \cdot ft^2 \cdot ^\circ F}{Btu}$), and $3.5in$ brick ($k = 6\frac{Btu \cdot in}{hr \cdot ft^2 \cdot ^\circ F}$). The inside surface of the plasterboard is maintained at $72^\circ F$. The outside film coefficient is $1.5\frac{Btu}{hr \cdot ft^2 \cdot ^\circ F}$ and the outside temperature is $20^\circ F$. What is the temperature at the interface of the brick and the insulation?

A. $24^\circ F$

B. $36^\circ F$

C. $56^\circ F$

D. $68^\circ F$

Find the total resistance for the Composite Wall, accouting for all resistances in series including the plasterboard, insulation, brick, and outside film coefficient. Note the inside surface is maintained at a specific temperature, so there is no need to account for an inside film coefficient. Make sure the units for each term are the same before adding.

$$R_{total} = R_{plasterboard} + R_{insulation} + \frac{L_{brick}}{k_{brick}} + \frac{1}{h_o}$$

$$R_{total} = 0.56\frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} + 13\frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} + \frac{3.5in}{6\frac{Btu \cdot in}{hr \cdot ft^2 \cdot ^\circ F}} + \frac{1}{1.5\frac{Btu}{hr \cdot ft^2 \cdot ^\circ F}} = 14.8\frac{hr \cdot ft^2 \cdot ^\circ F}{Btu}$$

Write an expression for the total heat flux through the wall. The total heat transfer, $\dot{Q}$, cannot be determined without the area of the wall being known, but the heat transfer per unit area i.e. heat flux, $\dot{q}$, can be found.

$$\dot{Q} = UA\Delta T$$

$$\frac{\dot{Q}}{A} = \dot{q} = U\Delta T$$

The overall heat transfer coefficient, U, is the inverse of the total resistance, R_{total}. $U = \frac{1}{R_t}$. Express heat flux in terms of total resistance. Substitute and solve.

$$\dot{q} = \frac{\Delta T}{R_t} = \frac{72^\circ F - 20^\circ F}{14.8\frac{hr \cdot ft^2 \cdot ^\circ F}{Btu}} = 3.5\frac{Btu}{hr \cdot ft^2}$$

The heat flux is the rate at which heat is conducted through the entire composite wall based on the total resistance of the wall. However, heat travels quickly through layers with minimal insulating properties and slowly through good insulators. Therefore, the temperature gradient throughout the composite wall is not uniform. To find the temperature at the interface of any two layers of the wall, find the resistance of all layers on one side of the interface. In this case, consider the interface between the insulation and the brick as location X. Find the combined resistance of the plasterboard and insulation.

$$R_{pb+ins} = R_{plasterboard} + R_{insulation} = 0.56\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu} + 13\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu} = 13.56\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu}$$

Using the previously determined heat flux for the entire wall, but only the partial resistance of the layers to the left of location X, find the temperature differential between the inside wall and location X. Note this is not the same as the previous delta T which is for the entire wall.

$$\dot{q} = \frac{\Delta T}{R_t}$$

$$\Delta T = \dot{q}R_{pb+ins} = \left(3.5\frac{Btu}{hr \cdot ft^2}\right)\left(13.56\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu}\right) = 47.6^\circ F$$

Solve for the temperature at location X.

$$\Delta T = 72^\circ F - T_x = 47.6^\circ F$$

$$T_x = 72^\circ F - 47.6^\circ F = 24.4^\circ F$$

Answer A