Find the forcing frequency, which is a function of the rotational speed.

$$\omega = \left(1800\frac{rev}{min}\right)\left(\frac{1min}{60s}\right)\left(\frac{2\pi rad}{rev}\right) = 188.5\frac{rad}{s}$$

Find the frequency ratio, r.

$$r = \frac{\omega}{\omega_n} = \frac{55.6\frac{rad}{s}}{188.5\frac{rad}{s}} = 3.39$$

Determine the vibration transmissibility, $\frac{F_T}{F_o}$, where F_T is the transmitted force and F_o is the unbalanced force. The damping ratio is given as $\varsigma = 0.3$.

$$\frac{F_T}{F_o} = \left[\frac{1+(2\varsigma r)^2}{(1-r^2)^2+(2\varsigma r)^2}\right]^{\frac{1}{2}} = \left[\frac{1+[2\,(0.3)\,(3.39)]^2}{\left(1-(3.39)^2\right)^2+[2\,(0.3)\,(3.39)]}\right]^{\frac{1}{2}} = 0.72$$

Solve for the transmitted force.

$$F_T = (0.72)\,F_o = (0.72)\,(2000lb_f) = 1440lb_f$$

Answer D

45.7 A $1.5in$ diameter steel shaft ($E = 2.9\times 10^7 psi$) is $4ft$ long and supported by two frictionless bearings at its ends. A $200lb_m$ flywheel is mounted on the center of the shaft. The shaft weight is negligible and there is no damping. What is the critical speed of the shaft?

A. $4Hz$

B. $12Hz$

C. $78Hz$

D. $590Hz$

The critical speed depends on the linear natural frequency of the shaft which can be determined from the static deflection due to the mass of the flywheel modeled as a point load applied to the center of a simple beam.

Start by calculating the area moment of inertia for a the shaft.

$$I = \frac{\pi r^4}{4} = \frac{\pi\,(0.75in)^4}{4} = 0.2485in^4$$

Find the formula for the static deflection of a simple beam with a concentrated load at center. Calculate the maximum static deflection.

$$\delta_{st} = y = \frac{Pl^3}{48EI} = \frac{(200lb_f)\,(48in)^3}{48\left(2.9\times 10^7\frac{lb_f}{in^2}\right)(0.2485in^4)} = 0.064in$$

The undamped natural circular frequency can then be determined as a function of the static deflection.

$$\omega_n = \sqrt{\frac{g}{\delta_{st}}} = \sqrt{\frac{\left(32.2\frac{ft}{s^2}\right)\left(\frac{12in}{1ft}\right)}{0.064in}} = 77.7\frac{rad}{s}$$

Find the linear natural frequency.

$$f_n = \frac{\omega_n}{2\pi} = \frac{77.7\frac{rad}{s}}{2\pi} = 12.4Hz$$

Answer B

45.8 A $200lb_m$ compressor rotates at $1750rpm$ and generates a disturbing force due to unbalance during each rotation. The compressor is mounted on 4 identical, equally loaded springs at the corners of its base. What individual spring stiffness is required to limit the transmitted force to 70% of the disturbing force?

A. $10,000\frac{lb_f}{in}$

B. $15,000\frac{lb_f}{in}$

C. $17,000\frac{lb_f}{in}$

D. $41,000\frac{lb_f}{in}$

Start with the vibration transmissibility formula. The ratio of the transmitted force to the unbalanced force, $\frac{F_T}{F_o} = 0.7$. Since there is no mention of damping, set the damping ratio, $\varsigma = 0$. Solve for r, which is the ratio of the forcing frequency to the natural frequency.

$$\frac{F_T}{F_o} = \left[\frac{1 + (2\varsigma r)^2}{(1 - r^2)^2 + (2\varsigma r)^2}\right]^{\frac{1}{2}}$$

$$0.7 = \left[\frac{1}{(1 - r^2)^2}\right]^{\frac{1}{2}}$$

$$0.49 = \frac{1}{(1 - r^2)^2}$$

$$\left(1 - r^2\right)^2 = 2.04$$

$$\left(1 - r^2\right) = 1.43$$

$$r^2 = -0.43$$