The undamped natural circular frequency can then be determined as a function of the static deflection.

$$\omega_n = \sqrt{\frac{g}{\delta_{st}}} = \sqrt{\frac{\left(32.2\frac{ft}{s^2}\right)\left(\frac{12in}{1ft}\right)}{0.064in}} = 77.7\frac{rad}{s}$$

Find the linear natural frequency.

$$f_n = \frac{\omega_n}{2\pi} = \frac{77.7\frac{rad}{s}}{2\pi} = 12.4Hz$$

Answer B

45.8 A $200lb_m$ compressor rotates at $1750rpm$ and generates a disturbing force due to unbalance during each rotation. The compressor is mounted on 4 identical, equally loaded springs at the corners of its base. What individual spring stiffness is required to limit the transmitted force to 70% of the disturbing force?

A. $10,000\frac{lb_f}{in}$

B. $15,000\frac{lb_f}{in}$

C. $17,000\frac{lb_f}{in}$

D. $41,000\frac{lb_f}{in}$

Start with the vibration transmissibility formula. The ratio of the transmitted force to the unbalanced force, $\frac{F_T}{F_o} = 0.7$. Since there is no mention of damping, set the damping ratio, $\varsigma = 0$. Solve for r, which is the ratio of the forcing frequency to the natural frequency.

$$\frac{F_T}{F_o} = \left[\frac{1 + (2\varsigma r)^2}{(1 - r^2)^2 + (2\varsigma r)^2}\right]^{\frac{1}{2}}$$

$$0.7 = \left[\frac{1}{(1 - r^2)^2}\right]^{\frac{1}{2}}$$

$$0.49 = \frac{1}{(1 - r^2)^2}$$

$$\left(1 - r^2\right)^2 = 2.04$$

$$\left(1 - r^2\right) = 1.43$$

$$r^2 = -0.43$$

Although the Reference Handbook does not address this issue, it is valid and expected that it will be necessary to take the absolute value of the right side before taking the square root. Other references such as the Mechanical Engineering Reference Manual show a version of the vibration transmissibility formula which includes absolute value symbols for precisely this reason.

$$r^2 = |-0.43| = 0.43$$

$$r = 0.65$$

Determine the forcing frequency, which is a function of the unbalanced force being exerted as the compressor rotates. The Reference Handbook uses ω, however to avoid ambiguity it may be useful to use ω_f.

$$\omega_f = \left(1750\frac{rev}{min}\right)\left(\frac{1min}{60s}\right)\left(\frac{2\pi rad}{rev}\right) = 183.3\frac{rad}{s}$$

Since r is the ratio of the forcing frequency to the natural frequency, rearrange to calculate the natural frequency required to limit the transmissibility to 70%, or in other words, to drive the value of r to 0.65.

$$r = \frac{\omega_f}{\omega_n}$$

$$\omega_n = \frac{\omega_f}{r} = \frac{183.3\frac{rad}{s}}{0.65} = 278\frac{rad}{s}$$

Rearrange the formula for natural frequency to find the total spring constant, k_t.

$$\omega_n = \sqrt{\frac{kg_c}{m}}$$

$$k_t = \frac{\omega_n^2 m}{g_c} = \frac{\left(278\frac{rad}{s}\right)^2 (200lb_m)}{\left(32.2\frac{lb_m \cdot ft}{lb_f \cdot s^2}\right)\left(\frac{12in}{1ft}\right)} = 40,578\frac{lb_f}{in}$$

Since there are 4 springs in parallel, divide by 4 to find the individual spring constant.

$$k_s = \frac{k_t}{4} = \frac{40,578\frac{lb_f}{in}}{4} = 10,144\frac{lb_f}{in}$$

Answer A