45.15 A company purchases a factory for $1M. The factory will generate a gross annual revenue of $140K and incur annual expenses of $60K. The facility has an expected life of 20 years and no salvage value. Assuming straight-line depreciation and a 30% effective tax rate, what is the after tax rate of return if the company sells the factory for $800K and exits the investment after 5 years? Ignore tax implications of capital losses at the time of sale.

A. 1.5%

B. 3.5%

C. 5.5%

D. 7.5%

Draw a cash flow diagram or make a list of cash flows. The cash flow in year 0 is -$1M. The cash flow in year 5 is +$800K in addition to the net profit after tax.

To determine the net profit after tax for years 1 through 5, start by subtracting the expenses from the revenue.

$$Profit\,Before\,Tax = Revenue - Expenses = \$140K - \$60K = \$80K$$

Because there is depreciation, the taxes are not assessed against the $80K. The taxable amounnt is reduced by the depreciation. Essentially, depreciation is applied as an expense even though the cash left the business in year 0. For straight line deprecation of $1M of initial cost spread over 20 years:

$$D = \frac{\$1,000,000}{20} = \$50K$$

The taxable income is therefore:

$$Taxable\,Income = \$80K - \$50K = \$30K$$

The tax liability based on a 30% tax rate is:

$$Tax = (0.3)\,(\$30K) = \$9K$$

Determine the Net Profit After Tax:

$$Net\,Profit\,After\,Tax = Profit\,Before\,Tax - Tax = \$80K - \$9K = \$71K$$

Write an expression for the present value of the investment accounting for the initial cost, the net profit after tax for years 1 through 5, and the salvage value in year 5. Set the present value equal to zero. By definition, the interest rate that gives the investment a present value of zero is the rate of return.

$$PV = -\$1,000,000 + \$71,000\,(P/A, i, 5) + \$800,000\,(P/F, i, 5) = 0$$

The math required for a direct solution becomes complex, so it is faster and equally reliable to use trial an error and interpolation to determine the interest rate. Guesses are best made in the vicinity of the answer choices with a strong preference for values offered in the Economic Factor Tables.

Try $i = 6\%$. Refer to the relevant Factor Table in the Economic Analysis section.

$$(P/A, 6\%, 5) = 4.2124$$

$$(P/F, 6\%, 5) = 0.7473$$

$$PV = -\$1,000,000 + \$71,000\,(4.2124) + \$800,000\,(0.7473) = -\$103,080$$

A negative present value implies the future cash flows, which are positive, are being discounted too much, therefore the 6% interest rate is too high.

Try $i = 2\%$.

$$(P/A, 2\%, 5) = 4.7135$$

$$(P/F, 2\%, 5) = 0.9057$$

$$PV = -\$1,000,000 + \$71,000\,(4.7135) + \$800,000\,(0.9057) = \$59,220$$

A positive present value implies the future cash flows, which are positive, are not being discounted enough, therefore the 2% interest rate guessed is too low.

Make a table and interpolate between $i = 6\%$ and $i = 2\%$

Interest Rate [%]	Present Value [$]
2	59,220
i	0
6	−103,080

$$\frac{59,220 - 0}{59,220 - (-103,080)} = \frac{2 - i}{2 - 6}$$

$$0.3649 = \frac{2 - i}{2 - 6}$$

$$-1.46 = 2 - i$$

$$i = 3.46\%$$

Answer B