

**45.16** A project has an initial cost of \$200K and generates \$30K per year in net profits. After 15 years there is a salvage value of \$40K. What is the return on investment for the project?

- A. 13.3%
- B. 14.2%
- C. 15.1%
- D. 16.0%

Draw a cash flow diagram or make a list of cash flows.

For year 0 there is an initial cost of -\$200K.

For years 1 through 15 there is an annual profit of \$30K.

For year 15 there is a salvage value of \$40K.

Write an expressions for the present value of the project. Set the present value equal to zero. The return on investment (ROI) is the interest rate that gives the initiative a present value of zero.

$$PV = -\$200,000 + \$30,000 (P/A, i\%, 15) + \$40,000 (P/F, i\%, 15) = 0$$

The math involved in solving for the interest rate is challenging and time consuming, therefore a trial and error method is often the best way forward from this point. One way to get an approximate answer to look at the answer choices and notice they are all between 12% and 18%. Another way is to notice that the one time salvage value is much less significant (though not negligible) than the annual profit, so it can be temporarily ignored as a first pass.

$$PV = -\$200,000 + \$30,000 (P/A, i\%, 15) \approx 0$$

$$(P/A, i\%, 15) \approx \frac{\$200K}{\$30K} \approx 6.7$$

Referring to the  $i = 12\%$  Factor Table, the value for  $(P/A, i\%, 15)$  is 6.8109 which is slightly higher than 6.7. Since  $P/A$  gets smaller as interest rates increase, the ROI must be greater than 12%. Also, the salvage value (previously ignored) is positive and drives the present value up which means future cash flows will need to be discounted slightly more and the interest rate is going to be greater than 12%. This is about as far as qualitative judgment goes. To arrive at a precise answer, it is necessary to find the PV using interest rates of 12% and 18%, then interpolate.

$$PV_{i=12} = -\$200,000 + \$30,000 (P/A, 12\%, 15) + \$40,000 (P/F, 12\%, 15)$$

$$PV_{i=12} = -\$200,000 + \$30,000 (6.8109) + \$40,000 (0.1827) = \$11,635 > 0$$

Therefore,  $ROI > 12\%$

$$PV_{i=18} = -\$200,000 + \$30,000 (P/A, 18\%, 15) + \$40,000 (P/F, 18\%, 15)$$

$$PV_{i=18} = -\$200,000 + \$30,000 (5.0916) + \$40,000 (0.0835) = -\$43,912 < 0$$

Therefore,  $ROI < 18\%$   
 Interpolate and solve for  $i$ .

$i$ [%]	$PV$ [\$]
12	11,635
$i$	0
18	-43,912

$$\frac{i - 12}{18 - 12} = \frac{0 - 11,635}{-43,912 - 11,635} = 0.2095$$

$$i - 12 = 1.257$$

$$i = 13.3\%$$

**Answer A**

**45.17 A \$100,000 investment today pays back \$10,000 in years 1 through 9 and \$100,000 in year 10. What is the rate of return?**

- A. 7.8%
- B. 8.6%
- C. 9.4%
- D. 10.3%

The rate of return is the interest rate that makes the present value zero. Draw a cash flow diagram or make a list of cash flows.

There is an initial outlay of \$100,000 in year zero (negative).

In years 1-9 there is a return of \$10,000 per year.

In year 10, there is a return of the original \$100,000 principle.

Write an expressions for the present value and set it equal to zero.

$$PV = -\$100,000 + \$10,000 (P/A, i, 9) + \$100,000 (P/F, i, 10) = 0$$

Direct solutions are challenging when the interest rate is unknown. The fastest approach is to guess values for  $i$  in close proximity to the answer choices. It is also possible to try the answer choices by trial and error; however, since only certain integer percentages can be found in the **Economic Factor Tables**, this would require using the **Economic Factor Conversions** to derive the cash flow factors. This solution will describe the process without testing answer choices.

Assume  $i = 8\%$ .

$$PV = -\$100,000 + \$10,000 (6.2469) + \$100,000 (0.4632) = \$8,789 > 0$$

Since the present value is *greater* than zero, the interest rate must be *higher*, thereby discounting the future (positive) cash flows *more* and *reducing* the present value.

Assume  $i = 10\%$ .