39.21 An R-410 refrigeration cycle operates between $25.5psia$ and $135psia$ with $20°F$ of superheat and $20°F$ of subcooling. The compressor efficiency is 88%. What is the coefficient of performance?

A. 4.3

B. 4.8

C. 5.1

D. 5.3

In the reference handbook lookup Pressure Versus Enthalpy Curves for Refrigerant 410A and work around the cycle accounting for superheating, subcooling, and compressor efficiency. Find the enthalpy for each state.

State 1, Follow a horizontal line at the low pressure condition (evaporator) to the right until the boundary with the superheated region, and then continue horizontally to the right at constant pressure accounting for an additional $20°F$ of superheat.

The refrigerant temperature at $25.5psia$ is $-40°F$. There is *additional* superheating of $20°F$. To find the temperature at State 1 accounting for the superheating, add $20°F$ to $-40°F$, which gives a temperature of $-20°F$ at State 1. The enthalpy should be looked up for this value.

$$P_1 = 25.5psia$$

$$20°F\ of\ superheat$$

$$T_1 = -20°F$$

$$h_1 = 118\frac{Btu}{lb}$$

State 2(ideal), Under ideal conditions, we would assume the compression is isentropic, therefore $s_2 = s_1$ and we can follow a line of constant entropy. We will make this assumption initially; however we must later account for the *actual* efficiency of the compressor.

$$P_2 = 135psia$$

$$s_2 = s_1$$

$$h_2 = 138\frac{Btu}{lb}$$

State 3, Regardless of the exact location of State 2, we know the pressure on the high side of the cycle and we know there is $20^{\circ}F$ of subcooling. Therefore we can locate State 3 and determine the enthalpy. Move horizontally to the left along the condenser process line (high pressure), and continue past the saturation curve into the subcooled liquid region.

$$P_3 = 135psia$$

$$20^{\circ}F\ subcooling$$

$$h_3 = 20\frac{Btu}{lb}$$

To address the compressor efficiency, it is useful to consider the ideal compressor work first, $W = \dot{m}\left(h_2 - h_1\right)$. This quantity will be *less than* the actual compressor work, $W = \dot{m}\left(h_2^{'} - h_1\right)$. The efficiency is the ratio of ideal to actual, and we can use it to specify the actual enthalpy at State 2, $h_2^{'}$.

$$\eta_{compressor} = \frac{W_{ideal}}{W_{actual}} = \frac{\dot{m}\left(h_2 - h_1\right)}{\dot{m}\left(h_2^{'} - h_1\right)} = \frac{\left(h_2 - h_1\right)}{\left(h_2^{'} - h_1\right)}$$

$$.88 = \frac{138\frac{Btu}{lb} - 118\frac{Btu}{lb}}{h_2^{'} - 118\frac{Btu}{lb}} \rightarrow h_2^{'} = 140.7\frac{Btu}{lb}$$

Note the enthalpy $h_2^{'}$ is higher than $h_2$because it takes more work to achieve the high pressure condition when operating with lower efficiency.

To find the coefficient of performance for a refrigerator, use the formula below, substitute, and solve. Make sure the enthalpy for State 2 is *actual* i.e. $h_2^{'}$. Note $h_4 = h_3$ as the expansion process is assumed to be isenthalpic.

$$COP_R = \frac{Q_L}{W} = \frac{\dot{m}\left(h_1 - h_4\right)}{\dot{m}\left(h_2^{'} - h_1\right)} = \frac{\left(h_1 - h_3\right)}{\left(h_2^{'} - h_1\right)} = \frac{\left(118\frac{Btu}{lb} - 20\frac{Btu}{lb}\right)}{\left(140.7\frac{Btu}{lb} - 118\frac{Btu}{lb}\right)} = 4.3$$

Answer A