36.42 An exterior wall is made up of $4inch$ brick cladding $\left(k = 0.42\frac{Btu \cdot ft}{hr \cdot ft^2 \cdot {}^\circ F}\right)$, $3\frac{1}{2}inch$ mineral fiber batt insulation $\left(C = 0.077\frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}\right)$, and $\frac{5}{8}inch$ gypsum board $\left(C = 1.78\frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}\right)$. The external surface of the brick is $0°F$ and the inside surface of the gypsum board is maintained at $70°F$. What is the temperature at the interface between the brick and the insulation?

A. $4°F$

B. $6°F$

C. $9°F$

D. $13°F$

Treat the problem as a Composite Wall with conduction through three layers: brick, insulation, and gypsum board. It is often necessary to look up the Thermal Resistance of Building Materials in the Reference Handbook, however, in this case the Thermal Conductivity and Conductance values were given.

Find the total thermal resistance through the composite wall. Use the thermal conductivity and thickness for the brick as given. For the insulation and gypsum board, the conductance has been given, which already accounts for the thickness.

$$R_T = \frac{L}{k_{brick}} + \frac{1}{C_{insulation}} + \frac{1}{C_{gypsum}}$$

$$R_T = \frac{(4in)\left(\frac{1ft}{12in}\right)}{0.42\frac{Btu \cdot ft}{hr \cdot ft^2 \cdot {}^\circ F}} + \frac{1}{0.077\frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}} + \frac{1}{1.78\frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}} = 14.3\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu}$$

Determine the Overall Coefficient of Heat Transfer for the composite wall:

$$U = \frac{1}{R_T} = \frac{1}{14.3\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu}} \approx .07\frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}$$

Find the heat flux i.e. heat transfer per unit area through the entire wall based on the overall coefficient of heat transfer. Use the temperatures for the internal and external surfaces.

$$\dot{Q} = UA\Delta T$$

$$\dot{q} = \frac{\dot{Q}}{A} = U\Delta T = \left(.07\frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}\right)(70°F - 0°F) = 4.88\frac{Btu}{hr \cdot ft^2}$$

To find the temperature at the interface of the brick and the insulation, consider only the thermal resistance of the brick, and the external temperature, treating the temperature at the brick/insulation interface as unknown. Note the heat flux is the same through the entire composite wall.

$$R_{brick} = \frac{(4ft)\left(\frac{12in}{1ft}\right)}{0.42\frac{Btu\cdot ft}{hr\cdot ft^2\cdot {}^\circ F}} = .794\frac{hr\cdot ft^2\cdot {}^\circ F}{Btu}$$

$$U_{brick} = \frac{1}{R_{brick}} = \frac{1}{.794\frac{hr\cdot ft^2\cdot {}^\circ F}{Btu}} = 1.26\frac{Btu}{hr\cdot ft^2\cdot {}^\circ F}$$

$$\dot{q} = U\Delta T \to \Delta T = \frac{\dot{q}}{U_{brick}} = \frac{4.88\frac{Btu}{hr\cdot ft^2}}{1.26\frac{Btu}{hr\cdot ft^2\cdot {}^\circ F}} = 3.87^\circ F$$

$$\Delta T = T_2 - T_1 = T_2 - 0^\circ F = 3.87^\circ F$$

$$T_2 = 3.87^\circ F \approx 4^\circ F$$

Answer A