31.4 Atmospheric air at $77°F$ undergoes isentropic compression to $100psia$ and $470°F$. How much work is done to the system during the process?

A. $0.5\frac{Btu}{lb}$

B. $70\frac{Btu}{lb}$

C. $90\frac{Btu}{lb}$

D. $50,000\frac{Btu}{lb}$

For an Isentropic Process, there are (at least) two formulas that may be used to find the work done from one state to another, which can be shown to be equivalent:

$$w = \frac{P_2 v_2 - P_1 v_1}{1-k}$$

$$w = \frac{R(T_2 - T_1)}{1-k}$$

Using the ideal gas law and solving for specific volume:

$$PV = mRT \rightarrow Pv = RT \rightarrow v = \frac{RT}{P}$$

Substitute for specific volume in the first formula and cancel pressure:

$$w = \frac{P_2\left(\frac{RT_2}{P_2}\right) - P_1\left(\frac{RT_1}{P_1}\right)}{1-k} = \frac{R(T_2 - T_1)}{1-k}$$

For convenience, use the second equation which is a function of temperature only:

$$w = \frac{R(T_2 - T_1)}{1-k} = \frac{\left(53.35\frac{ft \cdot lb_f}{lb_m \cdot R}\right)[470°F - 77°F]}{1-1.4} = \frac{-52,416\frac{ft \cdot lb_f}{lb_m}}{778\frac{ft \cdot lb_f}{Btu}} = -67\frac{Btu}{lb_m}$$

Normally it would be required to change Farenheit to Rankine; however, since it is a temperature *difference*, the delta is unchanged.

Note the unit conversion from $ft \cdot lb_f$ to Btu.

Finally, note the negative sign of the answer which aligns with the question's implication that work is being done *to the system*. A positive value would be expected if work were being done *by the system*.

Answer B