31.7 Equal parts of $50°F$ water and $250°F$ steam, each at $5psig$, are added to a mixing chamber and thoroughly mixed. What is the quality of the resulting mixture?

A. 41%

B. 47%

C. 53%

D. 59%

Let State 1 represent the water, State 2 represent the steam, and State 3 represent the resulting mixture. Use the Properties of Saturated Water and Steam table to look up the temperature and enthalpy of saturated liquid water at $5psig$.

$$P_1 = 5psig \approx 20psia$$

$$T_1 = 50°F$$

$$T_{sat@20psia} = 227.9°F$$

$$h_{f@20psia} = 196.25\frac{Btu}{lb}$$

Since $T_1 < T_{sat}$, the water is compressed or sub-cooled. To find the enthalpy h_1, use the specific heat capacity of water to relate the difference in enthalpy between h_f and h_1 to the difference in temperature between T_{sat} and T_1.

$$\Delta h = c_p \Delta T$$

$$h_f - h_1 = c_p \left(T_{sat} - T_1\right)$$

$$196.25\frac{Btu}{lb} - h_1 = \left(1\frac{Btu}{lb \cdot °F}\right)(229.9°F - 50°F)$$

$$h_1 = 18.3\frac{Btu}{lb}$$

Since $T_2 > T_{sat}$, the steam at state 2 is superheated. Therefore, refer to the Properties of Superheated Steam table and find the $P = 20psia$ table. Look up the enthalpy at state 2. Interpolate as needed.

$$P_2 = 5psig \approx 20psia$$

$$T_2 = 250°F$$

$$h_2 = 1167.2\frac{Btu}{lb}$$

Water and steam are mixed in equal parts, so the mass drops out of the mixing calculation for the enthalpy at State 3 and simply becomes the average. Find h_3.

$$h_3 = \frac{mh_1 + mh_2}{2m} = \frac{h_1 + h_2}{2} = \frac{18.3\frac{Btu}{lb} + 1167.2\frac{Btu}{lb}}{2} = 592.8\frac{Btu}{lb}$$

To find the quality at State 3, return to the saturated steam table and note the latent heat of vaporization, h_{fg}. Use the equation for quality to determine χ_3.

$$h_{fg@p=20psia} = 959.94\frac{Btu}{lb}$$

$$\chi_3 = \frac{h_3 - h_f}{h_{fg}} = \frac{592.8\frac{Btu}{lb} - 196.25\frac{Btu}{lb}}{959.94\frac{Btu}{lb}} = 0.413$$

Answer A