31.14 What is the density of water vapor in $68°F$ air with 40% relative humidity?

A. $4.3 \times 10^{-4} \frac{lb}{ft^3}$

B. $1.1 \times 10^{-3} \frac{lb}{ft^3}$

C. $1.6 \times 10^{-2} \frac{lb}{ft^3}$

D. $7.2 \times 10^{-2} \frac{lb}{ft^3}$

Recall the definition of Relative Humidity defined as the ratio of the partial pressure of water vapor divided by the saturation pressure at a given temperature.

$$\phi = \frac{p_w}{p_{ws}}$$

Deploying the ideal gas law, infer that the same relationship holds for density as for pressure.

$$PV = mRT \rightarrow P = \rho RT \rightarrow \rho = \frac{P}{RT}$$

R and T are constants in this case. Thus, $\rho \propto P$. Update the relative humidity formula accordingly, and solve for ρ_w.

$$\phi = \frac{\rho_w}{\rho_{sat}}$$

$$\rho_w = \phi \rho_{sat}$$

Look up the specific volume of water vapor at $68°F$ using the Properties of Saturated Water and Steam table, and use it to calculate the density at saturation.

$$\rho_{sat} = \frac{1}{v_g} = \frac{1}{925.15 \frac{ft^3}{lb}} = 0.00108 \frac{lb}{ft^3}$$

Apply the relative humidity to find the density of water in air with 40% humidity.

$$\rho_w = \phi \rho_{sat} = (0.4) \left(0.00108 \frac{lb}{ft^3} \right) = 0.00043 \frac{lb}{ft^3}$$

Answer A