31.16 **$80°F$ atmospheric air enters a compressor at a velocity of $10\frac{ft}{s}$ through a $50in^2$ inlet and exits at a velocity of $5\frac{ft}{s}$ and a temperature of $400°F$. The compressor does $150hp$ of work on the air. What is the heat input to the air?**

A. $-450,000\frac{Btu}{hr}$

B. $-320,000\frac{Btu}{hr}$

C. $-190,000\frac{Btu}{hr}$

D. $-60,000\frac{Btu}{hr}$

Start by recalling the equations that govern Steady-Flow Systems.

$$\dot{m}_i\left(h_i+\frac{V_i^2}{2}+gZ_i\right)-\dot{m}_e\left(h_e+\frac{V_e^2}{2}+gZ_e\right)+\dot{Q}_{in}-\dot{W}_{out}=0$$

$$\dot{m}_i=\dot{m}_e$$

Since the mass flow rate in and out are the same, it is not necessary to write the subscripts; simply refer to the mass flow rate as $\dot{m}$. To determine the mass flow rate, use the velocity and area at the inlet condition to find the volume flow rate. Then apply the ideal gas law to produce an expression for the density. Be mindful of unit conversions along the way.

$$Q=VA=\left(10\frac{ft}{s}\right)\left(50in^2\right)\left(\frac{1ft^2}{144in^2}\right)=3.47\frac{ft^3}{s}$$

$$PV=mRT\rightarrow P=\rho RT\rightarrow\rho=\frac{P}{RT}$$

$$\dot{m}=\rho Q=\frac{PQ}{RT}=\frac{\left(14.7\frac{lb_f}{in^2}\right)\left(\frac{144in^2}{ft^2}\right)\left(3.47\frac{ft^3}{s}\right)}{\left(53.3\frac{ft\cdot lb_f}{lb_m\cdot°R}\right)(540°R)}=0.255\frac{lb_m}{s}$$

Look up the Air at Low Pressure tables and obtain the enthalpy for air at the inlet and outlet conditions based solely on temperature.

$$T_1=80°F$$

$$h_1\approx 129\frac{Btu}{lb}$$

$$T_2=400°F$$

$$h_2\approx 206\frac{Btu}{lb}$$

It is possible to calculate the impact of the slight velocity change, however, it turns out to be negligible which is not unusual for problems such as this. Conscious of time, it is useful to omit the velocity terms from the solution. Similarly, there is no reason to suspect any meaningful change in height through the compressor, so the 'Z' term should also be neglected. Solve the steady flow energy equation for $\dot{Q}_{in}$. Note the work term has a negative sign because work is done *on the air*. In order for this term to be positive work would need to be done *by the air*. Another way to rationalize the negative sign is to recognize that $\dot{W}_{out} = -\dot{W}_{in}$, and $\dot{W}_{in}$ is positive.

$$\dot{W}_{out} = -\dot{W}_{in} = -150hp\left(42.4\frac{Btu}{min \cdot hp}\right)\left(\frac{1min}{60s}\right) = -106\frac{Btu}{s}$$

$$\dot{Q}_{in} = \dot{W}_{out} + \dot{m}\left(h_e - h_i\right) = -106\frac{Btu}{s} + \left(0.255\frac{lb_m}{s}\right)\left(206\frac{Btu}{lb} - 129\frac{Btu}{lb}\right) = -89.4\frac{Btu}{s}$$

$$\dot{Q}_{in} = -89.4\frac{Btu}{s}\left(\frac{3600s}{1hr}\right) = -321,840\frac{Btu}{hr}$$

Answer B