31.22 Each stage of a two-stage air compressor has a pressure ratio of 3. Air enters the first stage at $68°F$ and atmospheric pressure. An intercooler is used between the two stages and supplies $68°F$ air to the second stage of the process after removing $50,000\frac{Btu}{hr}$ of heat. Assuming the compression and intercooling processes are ideal, what is the mass flow rate of air?

A. $20\frac{lb_m}{hr}$

B. $260\frac{lb_m}{hr}$

C. $1070\frac{lb_m}{hr}$

D. $3590\frac{lb_m}{hr}$

Consider the air enterting the first compressor to be State 1, the air leaving the first compressor and entering the intercooler to be State 2, the air leaving the intercooler and entering the second compressor to be State 3, and the air leaving the second compressor to be State 4. Make a table to organize the temperature and pressure data given and needed.

State	$T\,[°F]$	$P\,[psia]$
1	68	14.7
2	T_2	P_2
3	68	P_3
4	T_4	P_4

The pressure at State 2 can be determined from the pressure at State 1 and the pressure ratio given.

$$\frac{P_2}{P_1} = 3 \rightarrow P_2 = 3\,(14.7psia) = 44.1psia$$

The cooling process from State 2 to State 3 is constant pressure, therefore the pressure at State 3 can also be established.

$$P_3 = P_2 = 44.1psia$$

The pressure at State 4 can be determined from the pressure at State 3 and the pressure ratio given.

$$\frac{P_4}{P_3} = 3 \rightarrow P_4 = 3\,(44.1psia) = 132.3psia$$

The temperature at State 2 can be determined using the formula for ideal air in a Constant Entropy Processes. Absolute temperatures must be used.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = (528^{\circ} R)\,(3)^{\frac{1.4-1}{1.4}} = 722.7^{\circ} R = 262.7^{\circ} F$$

The heat extracted during the cooling process from $2 \rightarrow 3$ is given, and air is considered to be ideal with constant specific heat capacity. Solve for the mass flow rate.

$$\dot{Q}_{2-3} = \dot{m} c_p \Delta T$$

$$\dot{m} = \frac{\dot{Q}_{2-3}}{c_p \Delta T} = \frac{50,000 \frac{Btu}{hr}}{\left(0.24 \frac{Btu}{lb^{\circ} F}\right)(262.7^{\circ} F - 68^{\circ} F)} = 1070 \frac{lb_m}{hr}$$

Answer C