31.30 In a Brayton cycle, $70°F$ atmostpheric air enters the compressor with a pressure ratio of 5:1. After the combustor, the gas temperature is $1600°F$. The turbine has an expansion ratio of 1:3. Assuming air behaves as an ideal gas and isentropic compression and expansion, what is the thermal efficiency of the cycle?

A. 10%

B. 20%

C. 30%

D. 40%

Consider the air entering the compressor as State 1, the air exiting the compressor and enterting the combustor as State 2, the air leaving the combustor and entering the turbine as State 3, and the air exiting the turbine as State 4.

The temperatures at States 1 and 3 are given. Determine the temperatures at States 2 and 4 based on isentropic compression and expansion and known compression and expansion ratios for the compressor and turbine. Be sure to use absolute temperatures i.e. degrees Rankine. For ideal air, set $k = 1.4$.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 530°R\,(5)^{\frac{1.4-1}{1.4}} = 839.4°R = 379.4°F$$

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 2060°R \left(\frac{1}{3}\right)^{\frac{1.4-1}{1.4}} = 1505°R = 1045°F$$

The efficiency of a Brayton Cycle is the net work divided by the heat input. Enthalpies may be replaced with temperatures if the gas is considered ideal.

$$\eta_{Brayton} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{out,turbine} - \dot{W}_{in,compressor}}{\dot{Q}_{in,combustor}} = \frac{\dot{W}_{34} - \dot{W}_{12}}{\dot{Q}_{23}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)} = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_2)}$$

$$\eta_{Brayton} = \frac{(1600°F - 1045°F) - (379.4°F - 70°F)}{(1600°F - 379.4°F)} = 0.20$$

Answer B